Question Number	Scheme	Marks
4(a)	$\frac{4r+2}{r(r+1)(r+2)} \equiv \frac{A}{r}+\frac{B}{r+1}+\frac{C}{r+2} \rightarrow A=...$ or $B=...$ or $C=...$	M1
	$\frac{1}{r}+\frac{2}{(r+1)}-\frac{3}{(r+2)}$	A1 A1
		(3)

Notes

M1: Correct partial fractions method e.g. substitution or compares coefficients to obtain one of A, B or C for $\frac{A}{r}, \frac{B}{(r+1)}, \frac{C}{(r+2)}$

A1: 2 Correct fractions (or values)

A1: Fully correct **partial fractions**.
It is not for just the correct values unless the correct fractions are seen in (b).

Correct answer with no working scores full marks in (a)

NB use of e.g. $\frac{4r+2}{r(r+1)(r+2)} \equiv \frac{A}{r(r+1)}+\frac{B}{(r+1)(r+2)} \rightarrow \frac{1}{r(r+1)}+\frac{3}{(r+1)(r+2)}$

Scores no marks in part (a) unless the fractions are split further.

(b)	**Must have partial fractions of the form $\frac{A}{r}, \frac{B}{(r+1)}, \frac{C}{(r+2)}$ $A, B, C \neq 0$ to score the <u>first</u> M mark in (b)**	
	$\sum_{r=1}^{n} = \left(\frac{1}{1}+\frac{2}{2}-\frac{3}{3}\right)+\left(\frac{1}{2}+\frac{2}{3}-\frac{3}{4}\right)+.....+\left(\frac{1}{n-1}+\frac{2}{n}-\frac{3}{n+1}\right)+\left(\frac{1}{n}+\frac{2}{n+1}-\frac{3}{n+2}\right)$	M1
	$=\frac{1}{1}+\frac{2}{2}+\frac{1}{2}-\frac{3}{n+1}+\frac{2}{n+1}-\frac{3}{n+2}$	A1 A1
	$=\frac{5(n+1)(n+2)-2(n+2)-6(n+1)}{2(n+1)(n+2)}$	**d**M1
	$\frac{n(5n+7)}{2(n+1)(n+2)}$	A1
		(5)
		Total 8

Notes

M1: Attempts at least the first 2 groups of terms and the last 2 groups of terms **which may be implied by their fractions identified below provided all the necessary terms (algebraic and constant) are seen**
Allow other letters for n (most likely to be r) except for the final mark – see below
If terms are found beyond the limits of the summation e.g. $r = 0$, $r = n + 1$, these can be ignored for this mark as long as at least the terms for $r = 1, 2, n - 1$ and n are seen.
No cancelling needs to be seen at this stage.

A1: $\frac{1}{1}+\frac{2}{2}+\frac{1}{2}\left(=\frac{5}{2}\right)$ identified as the only constant terms.

A1: $-\frac{3}{n+1}+\frac{2}{n+1}-\frac{3}{n+2}$ oe e.g. $-\frac{1}{n+1}-\frac{1}{n+2}-\frac{2}{n+2}$ identified as the only algebraic terms.

dM1: Attempt common denominator from terms of the form $A, \frac{B}{n+1}, \frac{C}{n+2}$ only.
Must see $(n + 1)(n + 2)$ in the denominator and an unsimplified or simplified polynomial of order 2 in the numerator.
Depends on the first method mark.

A1: Correct expression. Must be in terms of n for this mark.

Allow recovery in (b) from e.g.

$$\frac{4r+2}{r(r+1)(r+2)} \equiv \frac{A}{r(r+1)}+\frac{B}{(r+1)(r+2)} \rightarrow \frac{1}{r(r+1)}+\frac{3}{(r+1)(r+2)}$$

$$\frac{4r+2}{r(r+1)(r+2)} \equiv \frac{1}{r(r+1)}+\frac{3}{(r+1)(r+2)} = \frac{1}{r}-\frac{1}{r+1}+\frac{3}{r+1}-\frac{3}{r+2} \textbf{ etc.}$$

Note that there will be other correct approaches in (b):

<u>Alternative 1:</u> using change of summation limits:

$$\sum_{r=1}^{n}\frac{1}{r}+\frac{2}{(r+1)}-\frac{3}{(r+2)}=\sum_{r=1}^{n}\frac{1}{r}+\sum_{r=1}^{n}\frac{2}{(r+1)}-\sum_{r=1}^{n}\frac{3}{(r+2)}=\sum_{r=1}^{n}\frac{1}{r}+\sum_{r=2}^{n+1}\frac{2}{r}-\sum_{r=3}^{n+2}\frac{3}{r}$$

$$=-2+\frac{2}{n+1}+3+\frac{3}{2}-\frac{3}{n+1}-\frac{3}{n+2}+\sum_{r=1}^{n}\frac{1}{r}+\sum_{r=1}^{n}\frac{2}{r}-\sum_{r=1}^{n}\frac{3}{r}=\frac{5}{2}-\frac{1}{n+1}-\frac{3}{n+2}\text{ etc.}$$

Score as follows:

M1: for an attempt to change all summations to $\sum\frac{\cdots}{r}$ and add/subtract terms to compensate.

A1: for $-2+3+\frac{3}{2}$ oe **A1**: for $\frac{2}{n+1}-\frac{3}{n+1}-\frac{3}{n+2}$ oe

M1A1: As main scheme.

If you are unsure if an attempt deserves credit use Review.

<u>Alternative 2:</u> grouping terms:

$$\sum_{r=1}^{n}\frac{1}{r}+\frac{2}{(r+1)}-\frac{3}{(r+2)}=\sum_{r=1}^{n}\frac{1}{r}-\frac{1}{(r+2)}+\sum_{r=1}^{n}\frac{2}{r+1}-\frac{2}{(r+2)}$$

$$=\left(1-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\ldots+\frac{1}{n-1}-\frac{1}{n+1}+\frac{1}{n}-\frac{1}{n+2}\right)+2\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots+\frac{1}{n}-\frac{1}{n+1}+\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$=\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)+2\left(\frac{1}{2}-\frac{1}{n+2}\right)$$

Score as follows:

M1: for an attempt to group terms and consider at least 2 terms at the start and at least 2 terms at the end for each group.

A1: for $1+\frac{1}{2}+\frac{2}{2}$ oe **A1**: for $-\frac{1}{n+1}-\frac{1}{n+2}-\frac{2}{n+2}$ oe

M1A1: As main scheme.

5. Given that

$$(2-x^2)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2=3y$$

(a) show that

$$\frac{d^3y}{dx^3}=\frac{1}{(2-x^2)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)$$

(5)

Given also that $y=3$ and $\frac{dy}{dx}=\frac{1}{4}$ at $x=0$

(b) obtain a series solution for y in ascending powers of x with simplified coefficients, up to and including the term in x^3

(4)

(Total for Question 5 is 9 marks)

Note that part (a) is now being marked as MdMAddMA not MMABA