4. (a) Express $\dfrac{4r+2}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Hence, using the method of differences, prove that

$$\sum_{r=1}^{n} \frac{4r+2}{r(r+1)(r+2)} = \frac{n(an+b)}{2(n+1)(n+2)}$$

where a and b are constants to be found.

(5)

(Total for Question 4 is 8 marks)

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5. Given that

$$(2-x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 5x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 3y$$

(a) show that

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = \frac{1}{(2-x^2)}\left(2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\frac{\mathrm{d}y}{\mathrm{d}x}\right) - 5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$$

(5)

Given also that $y = 3$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{4}$ at $x = 0$

(b) obtain a series solution for y in ascending powers of x with simplified coefficients, up to and including the term in x^3

(4)

(Total for Question 5 is 9 marks)

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6. (a) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6\frac{\mathrm{d}y}{\mathrm{d}x} + 8y = 2x^2 + x$$

(8)

(b) Find the particular solution of this differential equation for which

$y = 1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = 0$

(5)

(Total for Question 6 is 13 marks)
