Question Number	Scheme	Marks
5(a)	$\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\rightarrow -2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$ **or** $5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2\rightarrow 5\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1
	$\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\rightarrow -2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$ **and** $5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2\rightarrow 5\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	**d**M1
	$-2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+5\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=3\dfrac{\mathrm{d}y}{\mathrm{d}x}$	A1
	$\Rightarrow\left(2-x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-5\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2-5x\times 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+3\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=\dfrac{1}{\left(2-x^2\right)}\left(2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\left(1-5\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)-5\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)*$	**dd**M1A1*
		(5)

Notes (Now marked as MdMAddMA not MMABA)

M1: Differentiates $\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using the product rule to obtain $...x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$

May be implied by e.g. $\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $-x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\rightarrow -2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$

i.e. $x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\rightarrow ...x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+...x^2\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$

Or Differentiates $5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using the product rule to obtain $...\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+...x\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

It must be clear that the expressions are not simply a result of expanding the given answer.

dM1: Differentiates $\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using the product rule to obtain $...x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$

May be implied by e.g. $\left(2-x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $-x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\rightarrow -2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}-x^2\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$

i.e. $x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\rightarrow ...x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+...x^2\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$

And Differentiates $5x\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ using the product rule to obtain $...\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+...x\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$

Depends on the first method mark.

A1: Fully correct differentiation e.g. $-2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(2-x^2\right)\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}+5\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2+5x\times 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=3\dfrac{\mathrm{d}y}{\mathrm{d}x}$

ddM1: Rearranges to make $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$ the subject. **Depends on both previous method marks.**

A1*: Obtains the printed answer from **fully correct** working.

(a) Alternative 1: Makes $\frac{d^2y}{dx^2}$ the subject first and then quotient rule.

	$$\frac{d^2y}{dx^2}=\frac{3y-5x\left(\frac{dy}{dx}\right)^2}{\left(2-x^2\right)}$$	
	$$\frac{d^3y}{dx^3}=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)-\left[3y-5x\left(\frac{dy}{dx}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$$	M1**d**M1A1
	$$\frac{d^3y}{dx^3}=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)-\left[3y-5x\left(\frac{dy}{dx}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$$ $$=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)-\left[\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-5x\left(\frac{dy}{dx}\right)^2\right](-2x)}{\left(2-x^2\right)^2}$$ $$=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-10x\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)+2x\left(2-x^2\right)\frac{d^2y}{dx^2}}{\left(2-x^2\right)^2}$$ $$\frac{d^3y}{dx^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)\ *$$	**dd**M1A1*

Notes (Now marked as MdMAddMA not MMABA)

M1: Attempts the quotient rule on $\frac{3y-5x\left(\frac{dy}{dx}\right)^2}{\left(2-x^2\right)}$ Award for the form

$\frac{\left(2-x^2\right)\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'\pm\left(2-x^2\right)'\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)}{\left(2-x^2\right)^2}$ where $\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'$ and $\left(2-x^2\right)'$ are

attempts to differentiate these expressions and includes an attempt to differentiate $5x\left(\frac{dy}{dx}\right)^2$

using the product rule to obtain $...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}$

dM1: Complete attempt at the quotient rule to obtain:

$$\frac{d^3y}{dx^3}=\frac{\left[...\frac{dy}{dx}+...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)\pm...x\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]}{\left(2-x^2\right)^2}$$

A1: Fully correct differentiation in any form.

ddM1: Replaces $3y$ with $\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2$ and cancels through by $\left(2-x^2\right)$

A1*: Obtains the printed answer from **fully correct** working.

(a) Alternative 2: Makes $\frac{d^2y}{dx^2}$ the subject first and then product rule.

	$\frac{d^2y}{dx^2}=\frac{3y-5x\left(\frac{dy}{dx}\right)^2}{\left(2-x^2\right)}=\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)\left(2-x^2\right)^{-1}$	
	$\frac{d^3y}{dx^3}=\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)^{-1}+\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]\times-\left(2-x^2\right)^{-2}\times-2x$	M1**d**M1A1
	$\frac{d^3y}{dx^3}=\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)^{-1}+2x\left(2-x^2\right)^{-2}\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]$ $\frac{d^3y}{dx^3}=\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)^{-1}+2x\left(2-x^2\right)^{-2}\left[\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2-5x\left(\frac{dy}{dx}\right)^2\right]$ $=\frac{\left[3\frac{dy}{dx}-5\left(\frac{dy}{dx}\right)^2-10x\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)+2x\left(2-x^2\right)\frac{d^2y}{dx^2}}{\left(2-x^2\right)^2}$ $\frac{d^3y}{dx^3}=\frac{1}{\left(2-x^2\right)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)$ *	**dd**M1A1*

Notes (Now marked as MdMAddMA not MMABA)

M1: Attempts the product rule on $\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)\left(2-x^2\right)^{-1}$ Award for the form $\left(2-x^2\right)^{-1}\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'+\left(\left(2-x^2\right)^{-1}\right)'\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)$ where $\left(3y-5x\left(\frac{dy}{dx}\right)^2\right)'$ and $\left(\left(2-x^2\right)^{-1}\right)'$ are attempts to differentiate these expressions and includes an attempt to differentiate $5x\left(\frac{dy}{dx}\right)^2$ using the product rule to obtain $...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}$

dM1: Complete attempt at the product rule to obtain:
$\frac{d^3y}{dx^3}=\left[...\frac{dy}{dx}+...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}\right]\left(2-x^2\right)^{-1}+...x\left(2-x^2\right)^{-2}\left[3y-5x\left(\frac{dy}{dx}\right)^2\right]$

A1: Fully correct differentiation in any form.

ddM1: Replaces $3y$ with $\left(2-x^2\right)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2$ and cancels through by $\left(2-x^2\right)$ or equivalent work.

A1*: Obtains the printed answer from **fully correct** working.

(a) Alternative 3: Makes $\frac{d^2y}{dx^2}$ the subject first, splits and then uses the quotient rule on both terms.

	$\frac{d^2y}{dx^2}=\frac{3y}{(2-x^2)}-\frac{5x\left(\frac{dy}{dx}\right)^2}{(2-x^2)}$	
	$\frac{d^3y}{dx^3}=\frac{3\left(\frac{dy}{dx}\right)(2-x^2)-3y(-2x)}{(2-x^2)^2}-\frac{\left[5\left(\frac{dy}{dx}\right)^2+5x\times2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)\right](2-x^2)-5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$	M1**d**M1A1
	$=\frac{3\frac{dy}{dx}(2-x^2)-\left((2-x^2)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2\right)(-2x)}{(2-x^2)^2}$ $-\frac{\left[5\left(\frac{dy}{dx}\right)^2+5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)-5x\left(\frac{dy}{dx}\right)^2(-2x)}{(2-x^2)^2}$ $=\frac{3\frac{dy}{dx}(2-x^2)+2x(2-x^2)\frac{d^2y}{dx^2}-\left[5\left(\frac{dy}{dx}\right)^2+5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right](2-x^2)}{(2-x^2)^2}$ $=\frac{3\frac{dy}{dx}+2x\frac{d^2y}{dx^2}-\left[5\left(\frac{dy}{dx}\right)^2+5x\times2\frac{dy}{dx}\frac{d^2y}{dx^2}\right]}{(2-x^2)}$ $=\frac{1}{(2-x^2)}\left(2x\frac{d^2y}{dx^2}\left(1-5\frac{dy}{dx}\right)-5\left(\frac{dy}{dx}\right)^2+3\frac{dy}{dx}\right)$ *	**dd**M1A1*

Notes (Now marked as MdMAddMA not MMABA)

M1: Attempts the quotient rule on $\frac{3y}{(2-x^2)}$ **and** $\frac{5x\left(\frac{dy}{dx}\right)^2}{(2-x^2)}$

Award for the forms $\frac{(2-x^2)(3y)'\pm3y(2-x^2)'}{(2-x^2)^2}$ **and** $\frac{(2-x^2)\left(5x\left(\frac{dy}{dx}\right)^2\right)'\pm(2-x^2)'\left(5x\left(\frac{dy}{dx}\right)^2\right)}{(2-x^2)^2}$

where $(3y)'$, $\left(5x\left(\frac{dy}{dx}\right)^2\right)'$ and $(2-x^2)'$ are attempts to differentiate these expressions and includes an attempt to differentiate $5x\left(\frac{dy}{dx}\right)^2$ using the product rule to obtain $...\left(\frac{dy}{dx}\right)^2+...x\frac{dy}{dx}\frac{d^2y}{dx^2}$

dM1: Complete attempt at both quotient rules to obtain:

$$\frac{d^3y}{dx^3}=\frac{...\left(\frac{dy}{dx}\right)(2-x^2)\pm...xy}{(2-x^2)^2}-\frac{\left[...\left(\frac{dy}{dx}\right)^2+...x\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)\right](2-x^2)\pm...x^2\left(\frac{dy}{dx}\right)^2}{(2-x^2)^2}$$

A1: Fully correct differentiation in any form.

ddM1: Combines both fractions with a common denominator of $(2-x^2)^2$, replaces $3y$ with $(2-x^2)\frac{d^2y}{dx^2}+5x\left(\frac{dy}{dx}\right)^2$ and cancels through by $(2-x^2)$

A1*: Obtains the printed answer from **fully correct** working.

Other methods e.g. rearranges, splits and uses 2 products can be marked in a similar way.
If you are unsure if a particular approach deserves credit use Review.

(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{9}{2}$	B1
	$\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=\frac{1}{2}\left(-5\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\frac{1}{2}\left(-5\times\frac{1}{16}+\frac{3}{4}\right)=\frac{7}{32}$	M1
	$\left(y=\right)3+\frac{1}{4}x+\frac{9}{2}\frac{x^2}{2!}+\frac{7}{32}\frac{x^3}{3!}$	M1
	$\left(y=\right)3+\frac{1}{4}x+\frac{9}{4}x^2+\frac{7}{192}x^3$	A1
		(4)
		Total 9

Notes

B1: Correct value for $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

M1: Attempts to find a value for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$.

Working need not be seen/checked as long as a value is obtained and a value may just be written down.

M1: Correct application of Taylor's series using their values for the derivatives (accept 2! or 2 and 3! or 6)

A1: Correct simplified series. The "$y=$" is not required.

6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 2x^2 + x$$

(8)

(b) Find the particular solution of this differential equation for which

$y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

(Total for Question 6 is 13 marks)