| Question Number | Scheme | Marks |
|---|---|---|
| **6(a)** | $m^2-6m+8=0 \Rightarrow (m-2)(m-4)=0,\ m=2,4$ | M1 |
| | $(\text{CF}=)\quad Ae^{2x}+Be^{4x}$ | A1 |
| | $\text{PI}: (y=)\alpha x^2+\beta x+\gamma$ | B1 |
| | $y'=2\alpha x+\beta \quad y''=2\alpha$<br>$2\alpha-6(2\alpha x+\beta)+8(\alpha x^2+\beta x+\gamma)=2x^2+x$ | M1 |
| | $8\alpha=2,\ -12\alpha+8\beta=1,\ 2\alpha-6\beta+8\gamma=0 \Rightarrow \alpha=...,\ \beta=...,\ \gamma=...$ | **d**M1 |
| | $\alpha=\frac{1}{4},\ \beta=\frac{1}{2},\ \gamma=\frac{5}{16}$ | A1A1 |
| | $y=Ae^{2x}+Be^{4x}+\frac{1}{4}x^2+\frac{1}{2}x+\frac{5}{16}$ | A1ft |
| | | **(8)** |

**Notes**

M1: Attempts to solve $m^2-6m+8=0$ by any valid method including a calculator so may be implied by correct values or the correct CF

A1: Correct CF ("$y=$" not required)

B1: Correct form for the PI ("$y=$" not required)

M1: Differentiates their PI twice and attempts to substitute into the given differential equation.
There must be evidence of $x^n \to x^{n-1}$ at least once.
The PI must be a quadratic expression with at least 2 terms and at least 2 unknowns.

**d**M1: Compares coefficients and solves to find values for their $\alpha$, $\beta$ and $\gamma$ from a PI of $y=\alpha x^2+\beta x+\gamma,\ \alpha,\beta,\gamma\neq 0$.
You do not need to check their working and they may just write down values.
**Depends on the <u>previous M mark only</u>.**

A1: Any 2 values correct **following the award of both previous method marks.**

A1: All correct **following the award of both previous method marks.**

A1ft: A complete solution, follow through their CF and PI.
Must be $y=...$ or $y=...$ must be seen at some point for their GS e.g. they may start $y$ = PI + CF
**All 3 M marks must have been earned**.

| | | |
|---|---|---|
| **6(b)** | $y = A\mathrm{e}^{2x} + B\mathrm{e}^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$ | |
| | $1 = A + B + \frac{5}{16}$ | M1 |
| | $\frac{\mathrm{d}y}{\mathrm{d}x} = 2A\mathrm{e}^{2x} + 4B\mathrm{e}^{4x} + \frac{1}{2}x + \frac{1}{2}$ $\quad 0 = 2A + 4B + \frac{1}{2}$ | M1 |
| | $A = \frac{13}{8}$ $\quad B = -\frac{15}{16}$ oe | **dd**M1A1 |
| | $y = \frac{13}{8}\mathrm{e}^{2x} - \frac{15}{16}\mathrm{e}^{4x} + \frac{1}{4}x^2 + \frac{1}{2}x + \frac{5}{16}$ oe | A1ft |
| | | **(5)** |
| | | **Total 13** |

**Notes**

M1: Substitutes $y = 1$ and $x = 0$ in their GS from (a) to obtain an equation in 2 unknowns.

M1: Differentiates their GS and substitutes $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $x = 0$ to obtain another equation in 2 unknowns.
You do not need to be concerned about the detail of their differentiation provided there is an attempt and their derivative is not their original function.

**dd**M1: Solves their 2 equations in 2 unknowns to obtain a value for both constants. You do not need to check their working and they may just write down values.
**Depends on both previous M marks.**

A1: Both values correct from a correct GS and fully correct work.

A1ft: Correct particular solution. Follow through their $A$ and $B$ and their PI from (a).
Must be $y =$... or $y =$... must be seen at some point for their GS but do not penalise its omission if already penalised in part (a).
**Depends on all 3 previous M marks.**

**7.** (a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where $k$ is a constant to be found.

**(3)**

Given that $z = \cos\theta + i\sin\theta$, where $\theta$ is real,

(b) show that

(i) $z^n + \frac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \frac{1}{z^n} = 2i\sin n\theta$

**(3)**

(c) Hence show that

$$\cos^3\theta\sin^3\theta = \frac{1}{32}(3\sin 2\theta - \sin 6\theta)$$

**(4)**

(d) Use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\,d\theta$$

**(4)**

**(Total for Question 7 is 14 marks)**