Question Number	Scheme	Marks
7(a)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^2-\frac{1}{z^2}\right)^3=z^6-3z^2+\frac{3}{z^2}-z^{-6}$	M1A1
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	A1
		(3)
	(a) Alternative	
	$\left(z+\frac{1}{z}\right)^3=z^3+3z+\frac{3}{z}+\frac{1}{z^3},\ \left(z-\frac{1}{z}\right)^3=z^3-3z+\frac{3}{z}-\frac{1}{z^3}$	M1A1
	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^3+3z+\frac{3}{z}+\frac{1}{z^3}\right)\left(z^3-3z+\frac{3}{z}-\frac{1}{z^3}\right)=...$ $=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	A1

(a) Notes

M1: Writes $\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3$ as $\left(z^2-\frac{1}{z^2}\right)^3$ and expands to obtain at least 3 terms of the correct form e.g. 3 from $...z^6,\ ...z^2, ...z^{-2}, ...z^{-6}$

A1: For $\left(z^2-\frac{1}{z^2}\right)^3=z^6-3z^2+\frac{3}{z^2}-z^{-6}$

A1: $z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$ Condone $z^6-z^{-6}-3\left(z^2-z^{-2}\right)$

Correct answer with no errors seen. The "$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=$" does not need to be seen.

Alternative:

M1: Attempt to expand both cubic brackets to obtain at least 3 terms of the correct form in both cases e.g. 3 from $...z^3,\ ...z, ...z^{-1}, ...z^{-3}$

A1: Both expansions correct.

A1: Expands, collects terms and obtains the correct answer with no errors seen.

Condone $z^6-z^{-6}-3\left(z^2-z^{-2}\right)$ for $z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$

The "$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=$" does not need to be seen.

(b)(i)(ii)	$z^n=\cos n\theta+\mathrm{i}\sin n\theta$	B1
	$z^{-n}=\cos\left(-n\theta\right)+\mathrm{i}\sin\left(-n\theta\right)$ or $=\cos n\theta-\mathrm{i}\sin n\theta$	M1
	$z^n+\frac{1}{z^n}=2\cos n\theta^*,\ z^n-\frac{1}{z^n}=2\mathrm{i}\sin n\theta^*$	A1*
		(3)

Notes

B1: $z^n=\cos n\theta+\mathrm{i}\sin n\theta$ seen anywhere.

M1: States or uses $z^{-n}=\cos\left(-n\theta\right)+\mathrm{i}\sin\left(-n\theta\right)$ (with or without the brackets) or $z^{-n}=\cos n\theta-\mathrm{i}\sin n\theta$

A1*: Both correct results obtained following sight of $z^{-n}=\cos n\theta-\mathrm{i}\sin n\theta$

(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=(2\cos\theta)^3(2\mathrm{i}\sin\theta)^3$ oe e.g. $-2^6\mathrm{i}\cos^3\theta\sin^3\theta$	B1
	$z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$ oe	B1ft
	$(2\cos\theta)^3(2\mathrm{i}\sin\theta)^3=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$ $\Rightarrow 8\cos^3\theta\times(-8\mathrm{i}\sin^3\theta)=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$ $-64\mathrm{i}\sin^3\theta\cos^3\theta=2\mathrm{i}\sin 6\theta-6\mathrm{i}\sin 2\theta$	**d**M1
	$\cos^3\theta\sin^3\theta=\frac{1}{32}(3\sin 2\theta-\sin 6\theta)$ *	A1*
		(4)

(c) Notes

B1: Uses $z+\frac{1}{z}=2\cos\theta$ and $z-\frac{1}{z}=2\mathrm{i}\sin\theta$ to express $\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3$ correctly in terms of $\sin\theta$ and $\cos\theta$. May be implied by e.g. lhs = $\left(\frac{1}{2}\left(z+\frac{1}{z}\right)\right)^3\left(\frac{1}{2\mathrm{i}}\left(z-\frac{1}{z}\right)\right)^3$.

May be seen in a rearranged form e.g. $\frac{-1}{64\mathrm{i}}\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\cos^3\theta\sin^3\theta$

B1ft: Uses $z^6-\frac{1}{z^6}=2\mathrm{i}\sin 6\theta$ and $z^2-\frac{1}{z^2}=2\mathrm{i}\sin 2\theta$ to express their $z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$ correctly in terms of $\sin\theta$ and $\cos\theta$. Follow through their *k* from part (a).

dM1: Equates both expressions and attempts to simplify $2^3\times(2\mathrm{i})^3$ to obtain $\alpha\mathrm{i}$ which may be implied.
Depends on both B marks.

A1*: Reaches the printed answer with no errors.

(d)	$\int \cos^3\theta\sin^3\theta\,\mathrm{d}\theta = \int \frac{1}{32}\left(3\sin 2\theta - \sin 6\theta\right)\mathrm{d}\theta$ $= \frac{1}{32}\left[-\frac{3}{2}\cos 2\theta + \frac{1}{6}\cos 6\theta\right]$	M1A1
	$= \frac{1}{32}\left[-\frac{3}{2}\cos 2\theta + \frac{1}{6}\cos 6\theta\right]_0^{\frac{\pi}{8}}$ $= \frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) - \left(-\frac{3}{2} + \frac{1}{6}\right)\right] = \frac{1}{32}\left(\frac{4}{3} - \frac{5\sqrt{2}}{6}\right)$	**d**M1A1
		(4)
		Total 14

(d) Notes

M1: Substitutes and integrates to obtain $p\cos 2\theta + q\cos 6\theta$

A1: Fully correct integration including the $\frac{1}{32}$

dM1: Substitutes **both** the given limits and subtracts the correct way round.
Do not allow if working in decimals.

There must be some evidence of both trigonometric terms evaluated with $\theta = \frac{\pi}{8}$ to give an expression containing $\sqrt{2}$ and both trigonometric terms evaluated with $\theta = 0$ to give a rational number.

A1: Correct exact value with terms collected e.g. $\frac{1}{32}\left(\frac{4}{3} - \frac{5\sqrt{2}}{6}\right)$ or any exact equivalent e.g. $\frac{1}{24} - \frac{5\sqrt{2}}{192}$

8.

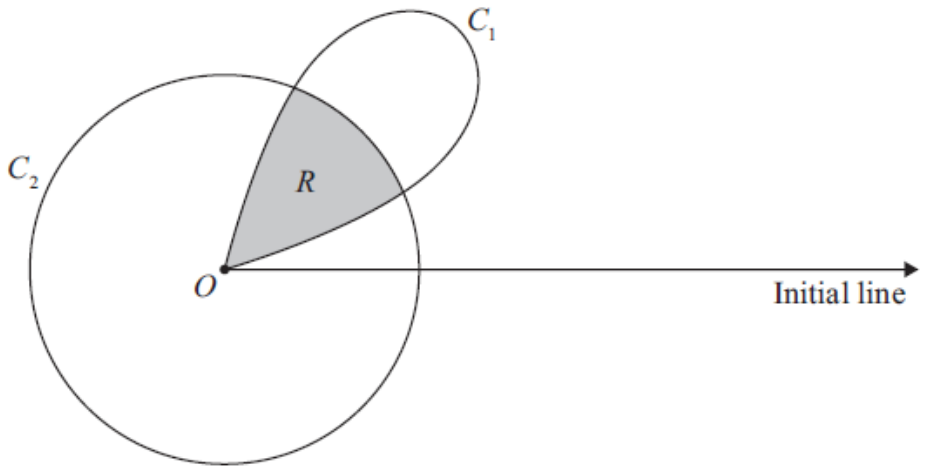


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows the curve C_1 with polar equation $r = 2a\sin 2\theta$, $0 \leqslant \theta \leqslant \frac{\pi}{2}$, and the circle C_2 with polar equation $r = a$, $0 \leqslant \theta \leqslant 2\pi$, where a is a positive constant.

(a) Find, in terms of a, the polar coordinates of the points where the curve C_1 meets the circle C_2

(3)

The regions enclosed by the curve C_1 and the circle C_2 overlap and the common region R is shaded in Figure 1.

(b) Use algebraic integration to find the area of the shaded region R, giving your answer in the form $\frac{1}{12}a^2\left(p\pi + q\sqrt{3}\right)$, where p and q are integers.

(7)

(Total for Question 8 is 10 marks)