**7.** (a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where $k$ is a constant to be found.

**(3)**

Given that $z = \cos\theta + \mathrm{i}\sin\theta$, where $\theta$ is real,

(b) show that

(i) $z^n + \dfrac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$

**(3)**

(c) Hence show that

$$\cos^3\theta\sin^3\theta = \frac{1}{32}\left(3\sin 2\theta - \sin 6\theta\right)$$

**(4)**

(d) Use algebraic integration to find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\,\mathrm{d}\theta$$

**(4)**

**(Total for Question 7 is 14 marks)**