Jun 2026
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1	$z_1 = \frac{1}{2}\left(\cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5}\right)$	$z_2 = \sqrt{3} - 3\mathrm{i}$	
(a)(i)	$\frac{1}{(z_1)^3} = (z_1)^{-3} = \left(\frac{1}{2}\right)^{-3}\left(\cos\frac{4\pi}{5} + \mathrm{i}\sin\frac{4\pi}{5}\right)^{-3}$ $= 8\left(\cos\frac{-12\pi}{5} + \mathrm{i}\sin\frac{-12\pi}{5}\right)$ or $8\mathrm{e}^{\frac{-12\pi}{5}\mathrm{i}}$	For 8... or ... $\left(\cos\frac{-12\pi}{5} + \mathrm{i}\sin\frac{-12\pi}{5}\right)$ oe etc	**M1**
	$8\mathrm{e}^{\frac{-2\pi}{5}\mathrm{i}}$	$8\mathrm{e}^{\frac{-2\pi}{5}\mathrm{i}}$ only	**A1**
(a)(ii)	$\lvert z_2\rvert = \sqrt{\left(\sqrt{3}\right)^2 + 3^2}$ $\left\{= \sqrt{12} = 2\sqrt{3}\right\}$	Correct numerical expression for $\lvert z_2\rvert$ or $\lvert z_2^*\rvert$	**M1**
	$\arg z_2 = -\arctan\left(\frac{3}{\sqrt{3}}\right) = -\arctan\left(\sqrt{3}\right) = -\frac{\pi}{3}$	A correct value in radians for a relevant angle. Implied by any of $\pm\frac{\pi}{3}$, $\pm\frac{\pi}{6}$, $\pm\frac{2\pi}{3}$ or $\pm\frac{5\pi}{6}$	**M1**
	$\frac{24\mathrm{e}^{\frac{4\pi}{5}\mathrm{i}}}{2\sqrt{3}\mathrm{e}^{-\frac{\pi}{3}\mathrm{i}}} = 4\sqrt{3}\mathrm{e}^{\frac{17\pi}{15}\mathrm{i}}$	Obtains $p\mathrm{e}^{q\mathrm{i}}$, using their z_1 and $2\sqrt{3}\mathrm{e}^{-\frac{\pi}{3}}$	**M1**
	$4\sqrt{3}\mathrm{e}^{-\frac{13\pi}{15}\mathrm{i}}$	Correct answer. Allow exact equivalents for $4\sqrt{3}$	**A1**
			(6)
(b)	$z = (z_1)^{\frac{1}{2}} = \frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)$	For $\frac{\sqrt{2}}{2}$ ... oe or ... $\left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)$ oe	**M1**
	$\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)$ or $\frac{\sqrt{2}}{2}\left(\cos\frac{7\pi}{5} + \mathrm{i}\sin\frac{7\pi}{5}\right)$	One correct root (any equivalents)	**A1**
	$\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)$ and $\frac{\sqrt{2}}{2}\left(\cos\frac{-3\pi}{5} + \mathrm{i}\sin\frac{-3\pi}{5}\right)$	Both correct roots in correct form. Allow exact equivalents for $\frac{\sqrt{2}}{2}$	**A1**
			(3)
			Total 9

Further Notes

Accept equivalent notations throughout (e.g. cos/sin form instead of exponential form for (a)(ii)) except for final answers.

(a)(i)

M1: A partial attempt at the cube, either the modulus or the argument sorted out correctly, though not necessarily simplified or in the correct range. You can condone if they miss an i for the method mark.

A1cao: Fully correct and simplified with argument in the specified range.

(a)(ii)
M1: Correctly attempts the modulus of z_2 or its conjugate (see alt below).
M1: Attempts the argument of z_2 or its conjugate. If no method shown accept any of the angles given, for method. Look for an attempt at an arctan of a relevant ratio.
M1: Look for an attempt at dividing moduli and subtracting argument for their attempts at z_1 and z_2 of suitable forms (Euler or cis form). Alternative here is to first multiply numerator and denominator by the conjugate of z_2 and then find in Euler form and multiply moduli, add arguments

$$\frac{48z_1}{\sqrt{3}-3\mathrm{i}} \times \frac{\sqrt{3}+3\mathrm{i}}{\sqrt{3}+3\mathrm{i}} = \frac{24\mathrm{e}^{\frac{4\pi\mathrm{i}}{5}}\left(2\sqrt{3}\mathrm{e}^{\frac{\pi}{3}\mathrm{i}}\right)}{12} = 4\sqrt{3}\mathrm{e}^{\frac{12\pi+5\pi}{15}\mathrm{i}}$$

A1: Correct answer with argument in the specified range. Allow simplified exact equivalents of $4\sqrt{3}$ e.g. $\frac{12}{\sqrt{3}}$

(b)
M1: Attempts the square root using De Moivre, with either a correct modulus or correct argument found.
A1: Any one correct answer in any form, need not be simplified.
A1: Both answer correct in the form specified and no other incorrect solutions extra solutions.

Note if they give $\pm\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$ and $\pm\frac{\sqrt{2}}{2}\left(\cos\frac{-3\pi}{5}+\mathrm{i}\sin\frac{-3\pi}{5}\right)$ then M1A1A1 can be scored since both correct roots are given in the correct form and no incorrect roots are given (there are repeated roots in an incorrect form). If just $\pm\frac{\sqrt{2}}{2}\left(\cos\frac{2\pi}{5}+\mathrm{i}\sin\frac{2\pi}{5}\right)$ is given it is M1A1A0.