| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| 7 | $y = z\cos x \qquad \cos^2 x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \sin 2x\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y = 12\cos 2x\cos^3 x$ | | |
| (a) | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \cos x\dfrac{\mathrm{d}z}{\mathrm{d}x} - z\sin x$ | Correct differentiation of $y = z\cos x$ | **B1** |
| | $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \cos x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2} - \sin x\dfrac{\mathrm{d}z}{\mathrm{d}x} - \sin x\dfrac{\mathrm{d}z}{\mathrm{d}x} - z\cos x$ | Attempts to differentiate a second time using the product rule at least once. | **M1** |
| | $\cos^2 x\left(\cos x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2} - 2\sin x\dfrac{\mathrm{d}z}{\mathrm{d}x} - z\cos x\right) + \sin 2x\left(\cos x\dfrac{\mathrm{d}z}{\mathrm{d}x} - z\sin x\right) + 2z\cos x = 12\cos 2x\cos^3 x$ | | **M1** |
| | Substitutes their derivatives into differential equation (I) | | |
| | $\cos^3 x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2} - 2\sin x\cos^2 x\dfrac{\mathrm{d}z}{\mathrm{d}x} - z\cos^3 x + 2\sin x\cos^2 x\dfrac{\mathrm{d}z}{\mathrm{d}x} - 2z\sin^2 x\cos x + 2z\cos x = 12\cos 2x\cos^3 x$<br>$\cos^3 x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2} - z\cos^3 x - 2z\cos x\left(1-\cos^2 x\right) + 2z\cos x = 12\cos 2x\cos^3 x$ | | **M1** |
| | Uses **correct** identities to obtain an equation in cos only | | |
| | $\cos^3 x\dfrac{\mathrm{d}^2z}{\mathrm{d}x^2} + z\cos^3 x = 12\cos 2x\cos^3 x$<br>$\Rightarrow \dfrac{\mathrm{d}^2z}{\mathrm{d}x^2} + z = 12\cos 2x$ * | Fully correct proof with no errors seen | **A1*** |
| | | | **(5)** |
| (b) | $z = \lambda\cos 2x,\ z' = -2\lambda\sin 2x,\ z'' = -4\lambda\cos 2x$<br>$-4\lambda\cos 2x + \lambda\cos 2x = 12\cos 2x$<br>$\Rightarrow -3\lambda = 12 \Rightarrow \lambda = -4$ | Uses PI of $z = \lambda\cos 2x$, differentiates twice to obtain correct forms and substitutes to obtain a value for $\lambda$ | **M1** |
| | $m^2 + 1 = 0 \Rightarrow m = \pm\mathrm{i}$<br>$\Rightarrow \mathrm{CF}: z = A\cos x + B\sin x$ | Correct CF. The "$z$ =" is not required | **B1** |
| | $\Rightarrow (z =)\ A\cos x + B\sin x - 4\cos 2x$ | Correct GS ft their CF and $\lambda$. The "$z$ =" is not required and condone "$y$=". | **M1** |
| | $\Rightarrow y = \cos x\left(A\cos x + B\sin x - 4\cos 2x\right)$ | Fully correct GS with "$y$ =" | **A1** |
| | | | **(4)** |
| (c) | $(0, 1) \Rightarrow 1 = 1(A + 0 - 4) \Rightarrow A = 5$ | Uses coordinates to obtain a value for $A$ | **M1** |
| | $\left(\dfrac{3\pi}{4}, 4\right) \Rightarrow 4 = -\dfrac{\sqrt{2}}{2}\left(-\dfrac{5\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}B - 0\right)$<br>$\Rightarrow 4 = \dfrac{5}{2} - \dfrac{1}{2}B \Rightarrow B = -3$ | Uses coordinates and their $A$ to obtain a value for $B$ | **dM1** |
| | $\left(\dfrac{\pi}{3}, p\right) \Rightarrow (p) = \dfrac{1}{2}\left(\dfrac{5}{2} - \dfrac{3\sqrt{3}}{2} + 2\right)$ | Uses coordinates and their $A$ and $B$ to obtain a numerical expression for $p$ | **ddM1** |
| | $(p) = \dfrac{1}{2}\left(\dfrac{9}{2} - \dfrac{3\sqrt{3}}{2}\right) = \dfrac{3}{4}\left(3 - \sqrt{3}\right)$ | Correct value for $p$ in the correct form | **A1** |
| | | | **(4)** |
| | | | **Total 13** |

**Further Notes**

Condone poor notation on the derivatives, use of *c*,*s* notation for cos*x* and sin*x* etc as long as the mathematics is correct.

(a) B0M1 is possible at the start for those do $\frac{dy}{dx} = \cos x \frac{dz}{dx}$ but must be producing a second derivative from their attempt at the first derivative, not just a general form. For the B1 they may first find $\frac{dy}{dz}$ then apply $\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$ but must obtain a correct expression involving $\frac{dy}{dx}$ to score the mark.

3rd M The trig identities identities use should be clear, no big jumps.

(b)

First M: The PI may include other terms (e.g. a $\mu \sin 2x$ term). Allow for $\cos 2x \rightarrow \ldots \sin 2x$ (where … can be any positive or negative number) as an attempt to differentate. They may must be substituting into the correct equation.

For the B and M marks, if their initial CF is incorrect (so B0) and when the forming the GS it is not is not consitent but still an attempt at a CF, award generously.

Alts:

(a) If they rearrange first they may get (oe) $z = \frac{y}{\cos x} \Rightarrow \frac{dz}{dx} = y\sec x \tan x + \frac{dy}{dx}\sec x$ which can score the B1, then for M1 the below (oe)

$$\frac{d^2z}{dx^2} = y\sec^3 x + y\sec x \tan^2 x + \frac{dy}{dx}\sec x \tan x + \frac{d^2y}{dx^2}\sec x + \frac{dy}{dx}\sec x \tan x$$

Then can substitute or adapt equation for the next M, e.g.

$$\Rightarrow \cos^3 x \frac{d^2z}{dx^2} = y + y\sin^2 x + 2\frac{dy}{dx}\sin x \cos x + \cos^2 x \frac{d^2y}{dx^2}$$

$$= \cos^2 x \frac{d^2y}{dx^2} + \sin 2x \frac{dy}{dx} + y + y(1 - \cos^2 x)$$

$$= \cos^2 x \frac{d^2y}{dx^2} + \sin 2x \frac{dy}{dx} + 2y - y\cos^2 x = 12\cos 2x \cos^3 x - z\cos^3 x$$

(both M's scored at this point as equation substituted and in terms of cos only)

$$\Rightarrow \frac{d^2z}{dx^2} = 12\cos 2x - z \Rightarrow \frac{d^2z}{dx^2} + z = 12\cos 2x$$

Variations on this theme are possible.

(c) If by error they only have one constant of integration in their equation, maximum M1dM0ddM0A0 can be scored for using either coordiante to find their constant.

An incorrect answer to (b) may mean they need to set and solve simultaneous equations to find their constants. M1 for full method to find one constant, dM1 for finding both using both coordinates.

(d)

ddMand A mark, condone *p* being called *y* for these marks.

Note if their answer to (c) is $y = A\cos x + B\sin x - 4\cos 2x$ then they can score M1dM1ddM1A0 in (d), they should get $A = 5$, $B = 5 + 4\sqrt{2}$ and $p = \frac{5}{2} + \left(5 + 4\sqrt{2}\right)\frac{\sqrt{3}}{2} + 2$ (though you are looking for the method, not these answers).