7. (a) Show that the substitution $y = z\cos x$, where z is a function of x, transforms the differential equation

$$\left(\cos^2 x\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\sin 2x\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \left(12\cos 2x\right)\cos^3 x \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}^2z}{\mathrm{d}x^2} + z = 12\cos 2x \qquad \text{(II)}$$

(5)

It is given that $z = \lambda\cos 2x$, where λ is a constant, is a particular integral of differential equation (II).

(b) By solving differential equation (II), determine the general solution of differential equation (I), giving the answer in the form $y = \mathrm{f}(x)$.

(4)

The graph of a particular solution to differential equation (I) passes through the points $(0, 1)$, $\left(\frac{3\pi}{4}, 4\right)$ and $\left(\frac{\pi}{3}, p\right)$ where p is a constant.

(c) Show that $p = q\left(3 - \sqrt{3}\right)$ where q is a rational number to be determined.

(4)

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Question 7 continued

(Total for Question 7 is 13 marks)