Question Number	Scheme	Notes	Marks
8(a)	$y=mx \quad y=\frac{1}{2}\left(x^2-4x+3\right)$ $x^2-4x+3=2mx$ $x^2-\left(4+2m\right)x+3=0$	Substitutes $y=mx$ into $y=\frac{1}{2}\left(x^2-4x+3\right)$ and rearranges to a 3TQ (the "= 0" is not required)	**M1**
	$d=\left(4+2m\right)^2-12=0$ $4+2m=\pm2\sqrt{3}\Rightarrow m=-2\pm\sqrt{3}$ **or** $4m^2+16m+4=m^2+4m+1=0$ $\Rightarrow\left(m+2\right)^2=3\Rightarrow m=-2\pm\sqrt{3}$ or $m=\frac{-4\pm\sqrt{16-4}}{2}=-2\pm\sqrt{3}$	M1: Applies discriminant = 0 and solves via usual rules to obtain a value for m A1: Both correct exact values.	**M1** **A1**
			(3)
(b)	E.g. $\arctan\left(-2+\sqrt{3}\right)=-15^\circ,-\frac{\pi}{12},-0.26...,-0.083\pi$ or $\arctan\left(-2-\sqrt{3}\right)=-75^\circ,-\frac{5\pi}{12},-1.3...,-0.42\pi$	Attempts to takes the arctan to find a relevant angle of one of their gradients.	**M1**
	$-15^\circ\leqslant\arg z\leqslant\ldots^\circ$ or $\ldots^\circ\leqslant\arg z\leqslant105^\circ$ $\left\{105^\circ=\frac{7\pi}{12},1.8...,0.58\pi\right\}$	One end of the range correct - value and direction of inequality (not just the value). Accept radians.	**A1**
	$-15^\circ\leqslant\arg z\leqslant105^\circ$	Fully correct range for arg z in degrees.	**A1**
			(3)
(c)	$\Rightarrow w=\mathrm{i}\left(x-2\mathrm{i}\right)^2+2y-5\mathrm{i}$ $=\mathrm{i}\left(x^2-4\mathrm{i}x-4\right)+x^2-4x+3-5\mathrm{i}$ $=\mathrm{i}x^2+4x-4\mathrm{i}+x^2-4x+3-5\mathrm{i}$	Substitutes $y=\frac{1}{2}\left(x^2-4x+3\right)$ and fully expands, condone a slip.	**M1**
	$u+\mathrm{i}v=x^2+3+\mathrm{i}\left(x^2-9\right)$ $\Rightarrow u=x^2+3,\quad v=x^2-9$ $v=u-12$	M1: Equates real and imaginary parts to obtain expressions in x for both u and v and eliminates x to get a linear equation. A1: Correct equation	**dM1** **A1**
			(3)
(d)	$\left\|w\right\|_{\min}=\frac{1}{2}\sqrt{12^2+12^2}$	Correct numerical expression for $\left\|w\right\|_{\min}$ Allow ft on $v=\pm u\pm k$	**M1**
	$6\sqrt{2}$	$6\sqrt{2}$ only	**A1cso**
			(2)
			Total 11
			PAPER TOTAL: 75

Further Notes

Alternative approaches to part (a):

Alt 1

M1: Finds the derivative at x, forms $y =$ "m"x with their derivative in terms of x and equates to the original curve.

$$\frac{\mathrm{d}y}{\mathrm{d}x}=x-2\Rightarrow y=(x-2)x\Rightarrow\frac{1}{2}(x^2-4x+3)=x(x-2)$$

M1: Expands and solves the resulting equation (usual rules – by calculator must be correct roots for their quadratic) and substitutes back into $m=x-2$ to find m.

A1: Correct values for m

(Note this is an informal way of doing Alt 2, but works since the equation $y = x^2 - 2x$ formed is representing the points that the line parallel to the tangent will pass through at a given value of x)

Alt 2

M1: Finds the derivative of the curve at a general point, $x = a$ say, and uses the gradient to set up the general equation of the tangent,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x - 2 \Rightarrow m = a - 2 \Rightarrow \text{tangent is } y - \frac{1}{2}(a^2 - 4a + 3) = (a - 2)(x - a)$$

M1: Sets the constant term from their tangent equation equal to 0, solves for a (usual rules– by calculator must be correct roots for their quadratic) and substitutes back in to find a value for m. A1 correct values

$$c = 0 \Rightarrow \frac{1}{2}(a^2 - 4a + 3) = (a - 2)a \Rightarrow a^2 = 3 \Rightarrow a = \pm\sqrt{3} \Rightarrow m = \pm\sqrt{3} - 2$$

(b)

M1: Correctly takes the arctan of one of their gradients. Could be in radians (awrt 2sf if necessary). Allow for evaluating any of $\arctan(\pm m)$ or $\arctan\left(\pm\frac{1}{m}\right)$. May be done via finding points on the tangents or the point on the curve that led to their m. Allow for any correct attempt to identify a relevant angle.

A1: A range with two endpoints and one correct and in the right place. Condone strict inequalities and radians. Degrees symbol not required.

A1: Degrees symbol not required. Any acceptable notation.

(c)

The substitution of y into the coordinates may be done after extracting real and imaginary terms, but the first M is not awarded until the substitution has been made. The dM is not awarded until x is eliminated and a a linear equation in u and v only is formed.

Alternative: Two points on C are e.g., $\left(0, \frac{3}{2}\right)$ and $(1, 0)$ and these transform to $(3, -9)$ and $(4, -8)$ (M1)

Equation of l is then, e.g., $\frac{v-(-9)}{-8-(-9)} = \frac{u-3}{4-3}$ (dM1) $\Rightarrow v = u - 12$ (A1)

(d)

May first find the closest point using $u = -v$ to solve and obtain $(6, -6)$, then find distance to origin.

Alt: uses $w = u + (u - 12)\mathrm{i}$ to find the magnitude and minimises.

$$|w| = \sqrt{u^2 + (u - 12)^2} = \sqrt{2u^2 - 24u + 144}$$
$$= \sqrt{2(u - 6)^2 - 72 + 144}$$
$$\Rightarrow \text{Mininum} = \sqrt{72} = 6\sqrt{2}$$

Final A is cso so must have come from the correct line.