Question Number	Scheme	Marks
5(a)	$w = 2 - \dfrac{3}{z+\mathrm{i}} \Rightarrow w = \dfrac{2z+2\mathrm{i}-3}{z+\mathrm{i}} \Rightarrow wz + w\mathrm{i} = 2z + 2\mathrm{i} - 3$ $\Rightarrow z(w-2) = 2\mathrm{i} - 3 - w\mathrm{i} \Rightarrow z = \dfrac{2\mathrm{i}-3-w\mathrm{i}}{w-2}$ oe e.g. $z = \dfrac{3}{2-w} - \mathrm{i}$	M1 A1
	$(z=)\dfrac{(v-3)+\mathrm{i}(2-u)}{(u-2)+\mathrm{i}v} \times \dfrac{(u-2)-\mathrm{i}v}{(u-2)-\mathrm{i}v} = \ldots$	dM1
	$(z=)\dfrac{(v-3)(u-2)-\mathrm{i}v(v-3)-\mathrm{i}(u-2)^2+v(2-u)}{(u-2)^2+v^2}$ $\Rightarrow x = \dfrac{6-3u}{(u-2)^2+v^2}$ and $y = \dfrac{\left(-u^2-v^2+4u+3v-4\right)}{(u-2)^2+v^2}$	ddM1
	Half-line $\arg z = \dfrac{\pi}{4} \Rightarrow y = x$	B1
	$6-3u = -u^2-v^2+4u+3v-4$ $\Rightarrow u^2+v^2-7u-3v+10=0$ $\Rightarrow \left(u-\dfrac{7}{2}\right)^2 + \left(v-\dfrac{3}{2}\right)^2 - \dfrac{49}{4} - \dfrac{9}{4} + 10 = 0$	dddM1
	$\Rightarrow \left(u-\dfrac{7}{2}\right)^2 + \left(v-\dfrac{3}{2}\right)^2 = \dfrac{9}{2}$	A1
		(7)
(b)(i)	radius is $\dfrac{3}{\sqrt{2}}$ or $\left(\dfrac{3\sqrt{2}}{2}\right)$	B1ft
(ii)	Centre is $\left(\dfrac{7}{2}, \dfrac{3}{2}\right)$	B1ft
		(2)
		Total 9

Notes(a)

M1: Completes an attempt to make z the subject.

A1: Correct expression. Any equivalent correct expression for z. Award when first seen and ISW if they then simplify incorrectly.

dM1: **Dependent on previous M mark.** Replaces w with $u + iv$ and shows an appropriate attempt to rationalise the denominator. Accept if $x + iy$ is used instead for this mark.

ddM1: **Dependent on both previous M marks.** Extracts the correct real and imaginary parts for their expression, allowing for sign errors only, but allow the 'i' coefficients in the imaginary parts.

B1: Any valid statement that $y = x$ on the half-line. This could be implied by setting their real and imaginary parts equal, after rationalisation, or equivalent work.

dddM1: **Dependent on all previous M marks** Attempts to equate their real and imaginary parts and proceed to the equation of a circle via a valid completing the square attempt. Must be a valid circle equation with equal coefficients for the square terms

A1: Correct circle equation obtained from correct work

(b)

In this part mark parts (i) and (ii) together but if the radius is clearly stated as a coordinate etc then B0

B1 ft: For correct radius ft their circle equation. Can only be scored if they have a valid circle equation

B1ft: For the centre ft their circle equation. Can only be scored if they have a valid circle equation

Note: Alternative methods are possible (see below). If in doubt send to review.

5(a) Alt 1	$z = x + iy, y = x \Rightarrow w = \dfrac{2z + 2i - 3}{z + i} = \dfrac{2x - 3 + i(2x + 2)}{x + (x+1)i}$	B1
	$\Rightarrow (u + iv)\left(x + (x+1)i\right) = 2x - 3 + i(2x + 2) \Rightarrow \ldots + \ldots i = \ldots$	M1
	$ux - v(x+1) + i\left(u(x+1) + vx\right) = (2x - 3) + i(2x + 2)$	A1
	$ux - v(x+1) = (2x-3) \Rightarrow x = \dfrac{v-3}{u-v-2}$ $u(x+1) + vx = (2x+2) \Rightarrow x = \dfrac{2-u}{u+v-2}$	dM1
	$\Rightarrow \dfrac{v-3}{u-v-2} = \dfrac{2-u}{u+v-2}$ $(v-3)(u+v-2) = (2-u)(u-v-2)$ $\Rightarrow uv + v^2 - 2v - 3u - 3v + 6 = 2u - 2v - 4 - u^2 + uv + 2u\,(=0)$ $\Rightarrow u^2 + v^2 - 7u - 3v + 10 = 0$	ddM1
	$\Rightarrow \left(u - \dfrac{7}{2}\right)^2 + \left(v - \dfrac{3}{2}\right)^2 - \dfrac{49}{4} - \dfrac{9}{4} + 10 = 0$	dddM1
	$\Rightarrow \left(u - \dfrac{7}{2}\right)^2 + \left(v - \dfrac{3}{2}\right)^2 = \dfrac{9}{2}$	A1
		(7)

5(a) ALT 2	$w = 2 - \dfrac{3}{z+\mathrm{i}} \Rightarrow w = \dfrac{2z+2\mathrm{i}-3}{z+\mathrm{i}} \Rightarrow wz + w\mathrm{i} = 2z + 2\mathrm{i} - 3$ $\Rightarrow z(w-2) = 2\mathrm{i} - 3 - w\mathrm{i} \Rightarrow z = \dfrac{2\mathrm{i}-3-w\mathrm{i}}{w-2}$ oe e.g. $z = \dfrac{3}{2-w} - \mathrm{i}$	M1A1
	$y = x \Rightarrow \lvert z-1\rvert = \lvert z-\mathrm{i}\rvert$	B1
	$\dfrac{\lvert 2\mathrm{i}-3-w\mathrm{i}-w+2\rvert}{\lvert w-2\rvert} = \dfrac{\lvert 2\mathrm{i}-3-w\mathrm{i}-\mathrm{i}(w-2)\rvert}{\lvert w-2\rvert}$ $\Rightarrow \lvert 2\mathrm{i}-3-(u+\mathrm{i}v)\mathrm{i}-(u+\mathrm{i}v)+2\rvert = \lvert 2\mathrm{i}-3-(u+\mathrm{i}v)\mathrm{i}-\mathrm{i}(u+\mathrm{i}v-2)\rvert$ $\Rightarrow \lvert(-1-u+v) + \mathrm{i}(2-u-v)\rvert = \lvert(-3+2v)+\mathrm{i}(4-2u)\rvert$	dM1
	$(-1-u+v)^2 + (2-u-v)^2 = (-3+2v)^2 + (4-2u)^2$ $\Rightarrow \cdots$	ddM1
	$1 - 2(v-u) + u^2 + v^2 - 2uv + 4 - 4u - 4v + u^2 + v^2 + 2uv = 9 - 12v + 4v^2 + 16 - 16u + 4u^2$ $\Rightarrow \cdots$ $\Rightarrow u^2 + v^2 - 7u - 3v + 10 = 0$ $\Rightarrow \left(u - \dfrac{7}{2}\right)^2 + \left(v - \dfrac{3}{2}\right)^2 - \dfrac{49}{4} - \dfrac{9}{4} + 10 = 0$	dddM1
	$\Rightarrow \left(u - \dfrac{7}{2}\right)^2 + \left(v - \dfrac{3}{2}\right)^2 = \dfrac{9}{2}$	A1
		(7)

Notes

(a) **Alt 1**

B1: Use of $y = x$ in the expression for z

M1: Uses $y = x$ and $z = x + iy$ in the expression for w and equates to $u + iv$, cross multiples and expands.

A1: Correct expression with the i^2 terms simplified.

dM1: Equates real and imaginary parts and proceeds to find two expressions for x. **Dependent on previous M mark.**

ddM1: Eliminate x by solving simultaneously then proceed to simplify by cross multiplying and expanding to at least eliminate uv terms. **Dependent on both previous M marks.**

dddM1: Dependent on all previous M marks proceeds to the equation of a circle via a valid competing the square attempt. Must be a valid circle equation with equal coefficients for the square terms

A1: Correct circle equation obtained from correct work

(a) **Alt 2**

M1: Completes an attempt to make z the subject.

A1: Correct expression. Any equivalent correct expression for z. Award when first seen and ISW if they then simplify incorrectly.

B1: Any valid statement that $y = x$ on the half-line. This could be implied by using $|z - 1| = |z - i|$

dM1: **Dependent on previous M mark.** Replaces w with $u + iv$ in their modulus equation, collects real and imaginary parts and equates the numerators

ddM1: **Dependent on both previous M marks.** Applies a correct method to find the modulus of both sides (with no i terms) etc and $i^2 = -1$ etc and proceeds to collect terms

dddM1: **Dependent on all previous M marks** Proceeds to the equation of a circle via a valid completing the square attempt. Must be a valid circle equation with equal coefficients for the square terms

A1: Correct circle equation obtained from correct work