Question Number	Scheme	Marks
6(a)	$z=\cos\theta+\mathrm{i}\sin\theta$ **and** $\frac{1}{z}=\cos\theta-\mathrm{i}\sin\theta$ $\Rightarrow z+\frac{1}{z}=\ldots$ or $\Rightarrow z-\frac{1}{z}=\ldots$	M1
	$\frac{1}{2}\left(z+\frac{1}{z}\right)=\cos\theta$ and $\frac{1}{2\mathrm{i}}\left(z-\frac{1}{z}\right)=\sin\theta$	A1*
		(2)
(b)	$-\frac{1}{8\mathrm{i}}\left(z^3+3z^2\left(\frac{-1}{z}\right)+3z\left(\frac{-1}{z}\right)^2-\frac{1}{z^3}\right)$ **OR** $\frac{1}{16}\left(z^4+4z^3\left(\frac{1}{z}\right)+6z^2\left(\frac{1}{z}\right)^2+4z\left(\frac{1}{z}\right)^3+\left(\frac{1}{z}\right)^4\right)$	M1
	$\frac{\mathrm{i}}{128}\left(z^3-3z+\frac{3}{z}-\frac{1}{z^3}\right)\left(z^4+4z^2+6+\frac{4}{z^2}+\frac{1}{z^4}\right)=\ldots$	M1
ALT for first two M marks	$\left(z^2-\frac{1}{z^2}\right)^3=z^6+3z^4\left(-\frac{1}{z^2}\right)+3z^2\left(-\frac{1}{z^2}\right)^2+\left(-\frac{1}{z^2}\right)^3$	M1
	$=\frac{\mathrm{i}}{128}\left(z^6+3z^4\left(-\frac{1}{z^2}\right)+3z^2\left(-\frac{1}{z^2}\right)^2+\left(-\frac{1}{z^2}\right)^3\right)\left(z+\frac{1}{z}\right)=\cdots$	M1
	$=\frac{\mathrm{i}}{128}\left[\left(z^7-\frac{1}{z^7}\right)+\left(z^5-\frac{1}{z^5}\right)-3\left(z^3-\frac{1}{z^3}\right)-3\left(z-\frac{1}{z}\right)\right]$	M1
	$=\frac{\mathrm{i}}{128}\left(2\mathrm{i}\sin7\theta+2\mathrm{i}\sin5\theta-6\mathrm{i}\sin3\theta-6\mathrm{i}\sin\theta\right)$	M1
	$=-\frac{1}{64}\sin7\theta-\frac{1}{64}\sin5\theta+\frac{3}{64}\sin3\theta+\frac{3}{64}\sin\theta$ o.e.	A1
		(5)

ALT	Use of $\sin^3\theta\cos^4\theta = \sin^3\theta\left(1-\sin^2\theta\right)^2 = \sin^3\theta - 2\sin^5\theta + \sin^7\theta$	
	$\sin^3\theta = -\frac{1}{8i}\left(z^3 + 3z^2\left(-\frac{1}{z}\right) + 3z\left(-\frac{1}{z}\right)^2 - \frac{1}{z^3}\right) = \underline{-\frac{1}{8i}\left(2i\sin 3\theta - 6i\sin\theta\right)}$ **Or** $\sin^5\theta = \frac{1}{32i}\left(z^5 + 5z^4\left(-\frac{1}{z}\right) + 10z^3\left(-\frac{1}{z}\right)^2 + 10z^2\left(-\frac{1}{z}\right)^3 + 5z\left(-\frac{1}{z}\right)^4 - \frac{1}{z^5}\right)$ $= \underline{\frac{1}{32i}\left(2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta\right)}$ **Or** $\sin^7\theta = \frac{i}{128}\left(z^7 + 7z^6\left(-\frac{1}{z}\right) + 21z^5\left(-\frac{1}{z}\right)^2 + 35z^4\left(-\frac{1}{z}\right)^3 + 35z^3\left(-\frac{1}{z}\right)^4 + 21z^2\left(-\frac{1}{z}\right)^5 + 7z\left(-\frac{1}{z}\right)^6 - \frac{1}{z^7}\right)$ $= \underline{\frac{i}{128}\left(2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta\right)}$	M1
	$\sin^3\theta = -\frac{1}{8i}\left(z^3 + 3z^2\left(-\frac{1}{z}\right) + 3z\left(-\frac{1}{z}\right)^2 - \frac{1}{z^3}\right) = \underline{-\frac{1}{8i}\left(2i\sin 3\theta - 6i\sin\theta\right)}$ **and** $\sin^5\theta = \frac{1}{32i}\left(z^5 + 5z^4\left(-\frac{1}{z}\right) + 10z^3\left(-\frac{1}{z}\right)^2 + 10z^2\left(-\frac{1}{z}\right)^3 + 5z\left(-\frac{1}{z}\right)^4 - \frac{1}{z^5}\right)$ $= \underline{\frac{1}{32i}\left(2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta\right)}$ **and** $\sin^7\theta = \frac{i}{128}\left(z^7 + 7z^6\left(-\frac{1}{z}\right) + 21z^5\left(-\frac{1}{z}\right)^2 + 35z^4\left(-\frac{1}{z}\right)^3 + 35z^3\left(-\frac{1}{z}\right)^4 + 21z^2\left(-\frac{1}{z}\right)^5 + 7z\left(-\frac{1}{z}\right)^6 - \frac{1}{z^7}\right)$ $= \underline{\frac{i}{128}\left(2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta\right)}$	M1
	$\therefore \sin^3\theta\cos^4\theta = -\frac{1}{8i}\left(2i\sin 3\theta - 6i\sin\theta\right) - 2\left(\frac{1}{32i}\left(2i\sin 5\theta - 10i\sin 3\theta + 20i\sin\theta\right)\right)$ $+\frac{i}{128}\left(2i\sin 7\theta - 14i\sin 5\theta + 42i\sin 3\theta - 70i\sin\theta\right) = \cdots$	M1
	$= \frac{\mathrm{i}}{128}\left(2\mathrm{i}\sin 7\theta + 2\mathrm{i}\sin 5\theta - 6\mathrm{i}\sin 3\theta - 6\mathrm{i}\sin\theta\right)$	M1
	$= -\frac{1}{64}\sin 7\theta - \frac{1}{64}\sin 5\theta + \frac{3}{64}\sin 3\theta + \frac{3}{64}\sin\theta$	A1
		(5)

(c)	$\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^4\theta \,\mathrm{d}\theta = -\frac{1}{64}\int_0^{\frac{\pi}{2}} \left(\sin 7\theta + \sin 5\theta - 3\sin 3\theta - 3\sin\theta\right)\mathrm{d}\theta$ $= -\frac{1}{64}\left[\frac{-\cos 7\theta}{7} \frac{-\cos 5\theta}{5} + \cos 3\theta + 3\cos\theta\right]_0^{\frac{\pi}{2}}$	M1A1ft
	$-\frac{1}{64}\left[\left(0+0+0+0\right)-\left(-\frac{1}{7}-\frac{1}{5}+1+3\right)\right]$	M1
	$=\frac{2}{35}$	A1
		(4)
		Total 11

Notes

(a)

M1: Uses de Moivre correctly to obtain correct expression for $\frac{1}{z}$ and <u>either</u> adds or subtracts to *z.* Must connect LHS and RHS for the first or the second expression in some way for one of the expressions- a collection of trig terms with no connection to *z* and $\frac{1}{z}$ is M0

A1*: Both correct expressions obtained from correct work

(b)

Note: this is a 'hence' question so candidates must use the identities in (a)

M1: Attempts to expand <u>either</u> bracket using binomial expansion, coefficients must be correct but condone slips if the intention is clear

M1: Attempts to <u>multiply</u> <u>both expanded</u> brackets, in terms of *z*, together

<u>ALT for first two M marks</u>

M1: (Recognising the product can be written as a difference of two squares)

Expands $\left(z^2-\frac{1}{z^2}\right)^3$. Coefficients must be correct but condone slips if the intention is clear

M1: Attempts to multiply their expansion of $\left(z^2-\frac{1}{z^2}\right)^3$ with $\left(z+\frac{1}{z}\right)$

M1: Groups terms in *z* correctly to form only sine terms after multiplication - could be implied by correct sine terms seen

M1: Use of general De Moivre $\left(z^n-\frac{1}{z^n}\right)\equiv 2\mathrm{i}\sin n\theta$ to obtain at least one correct term on the RHS- but the i's must be dealt with correctly- either consistently cancelled or remain in place

A1: Correct final result obtained from correct work.

(b)

ALT

M1: Any of these expansions in terms of z or in terms of sin terms. Coefficients must be correct but condone slips if the intention is clear

M1: All three of these expansions in terms of z or in terms of sin terms. Coefficients must be correct but condone slips if the intention is clear

M1: Substitutes into $\sin^3\theta - 2\sin^5\theta + \sin^7\theta$ and attempts to collect terms

M1: Obtains at least one correct term on the RHS

A1: Correct final result obtained from correct work

(c)

M1: Integrates an expression of the form $a\sin 7\theta + b\sin 5\theta + c\sin 3\theta + d\sin\theta$ to obtain an expression of the form $...\cos 7\theta + ...\cos 5\theta + ...\cos 3\theta + ...\cos\theta$ but M0 if they are clearly differentiating. Accept if the candidate integrates with coefficients as *a, b, c, d*

A1ft: Correct integration ft their values from (b) and accept in terms of *a, b, c, d*

M1: Substitutes the correct limits and subtracts the correct way round. This can be implied by a correct final answer, once the first M1A1 is scored as no calculator warning given in this question. M0 if they have clearly differentiated.

A1: Fully correct result, from correct integration