Question Number	Scheme	Marks
8		
(a)	POI: $2\cos\left(\theta+\frac{\pi}{6}\right)=\sec\left(\theta-\frac{\pi}{3}\right)\Rightarrow 2\cos\left(\theta+\frac{\pi}{6}\right)\cos\left(\theta-\frac{\pi}{3}\right)=1$	B1
	$2\cos\left(\theta+\frac{\pi}{6}\right)\sin\left(\theta+\frac{\pi}{6}\right)=1\Rightarrow \sin\left(2\theta+\frac{\pi}{3}\right)=1\Rightarrow 2\theta=\cdots\Rightarrow\theta=\cdots$	M1
	$2\theta=\frac{\pi}{6},\ldots\Rightarrow\theta=\frac{\pi}{12}$ and $r=2\cos\frac{\pi}{4}=\sqrt{2}$	A1
		(3)
ALT	Use of $\cos\theta\equiv-\cos(\theta+\pi)$	
	$2\cos\left(\theta+\frac{\pi}{6}\right)=-\sec\left(\theta+\frac{2\pi}{3}\right)\Rightarrow 2\cos\left(\theta+\frac{\pi}{6}\right)\cos\left(\theta+\frac{2\pi}{3}\right)=-1$	B1
	$2\sin\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{2\pi}{3}\right)=-1\Rightarrow \sin\left(2\theta+\frac{4\pi}{3}\right)=-1\Rightarrow 2\theta=\cdots\Rightarrow\theta=\cdots$	M1
	$2\theta=\frac{3\pi}{2}-\frac{4\pi}{3}=\frac{\pi}{6}\Rightarrow\theta=\frac{\pi}{12}$ and $r=2\cos\frac{\pi}{4}=\sqrt{2}$	A1
		(3)
(b) Way 1	**<u>Way1</u>** $\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{12}}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2\mathrm{d}\theta-\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\mathrm{d}\theta$ Overall strategy	M1
	$\left(\frac{1}{2}\right)\int\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2\mathrm{d}\theta=k\tan\left(\theta-\frac{\pi}{3}\right)$ **OR** $\left(\frac{1}{2}\right)\int\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	For **both** $\left(\frac{1}{2}\right)\int\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2\mathrm{d}\theta=k\tan\left(\theta-\frac{\pi}{3}\right)$ **and** $\left(\frac{1}{2}\right)\int\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1

	For each correct integral $\left(\frac{1}{2}\right)\left[\tan\left(\theta-\frac{\pi}{3}\right)\right]-\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$	A1 A1
	$\frac{1}{2}\left[\tan\left(\theta-\frac{\pi}{3}\right)\right]_0^{\frac{\pi}{12}}-\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]_0^{\frac{\pi}{12}}$ $=\frac{1}{2}\left(\sqrt{3}-1\right)-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]=\ldots$	dM1
	$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$	A1
		(7)
8(b) **Way 2**	small triangle $-\left(\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\,\mathrm{d}\theta-\text{large triangle}\right)$ Overall strategy mark	M1
	Area of small triangle $=\frac{1}{2}\left[2-\sqrt{2}\,\cos\frac{\pi}{12}\right]\left[\sqrt{2}\sin\frac{\pi}{12}\right]=\ldots$	M1
	$=\left[\frac{2\sqrt{3}-3}{4}\right]$	A1
	$\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\,\mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	$\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$	A1
	Area = $=\left[\frac{2\sqrt{3}-3}{4}\right]-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]+\frac{1}{2}\sqrt{2}\,\cos\frac{\pi}{12}\sqrt{2}\,\sin\frac{\pi}{12}=\cdots$	dM1
	$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$	A1
		(7)

8(b) **Way 3**	Largest triangle - $\int_0^{\frac{\pi}{12}}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\,\mathrm{d}\theta$ Overall strategy mark	M1
	Largest triangle $\frac{1}{2}\times 2\times\sqrt{2}\sin\frac{\pi}{12}\quad=\ldots$	M1
	$=\frac{\sqrt{3}-1}{2}$	A1
	$\left(\frac{1}{2}\right)\int\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2\,\mathrm{d}\theta=k\left[\theta+\frac{\sin\left(2\theta+\frac{\pi}{3}\right)}{2}\right]$	M1
	$\left(\frac{1}{2}\right)\left[2\theta+\sin\left(2\theta+\frac{\pi}{3}\right)\right]$	A1
	Area = $=\frac{\sqrt{3}-1}{2}-\left[\left(\frac{\pi}{12}+\frac{1}{2}\right)-\left(0+\frac{\sqrt{3}}{4}\right)\right]=\cdots$	dM1
	$=\frac{3}{4}\sqrt{3}-1-\frac{\pi}{12}$	A1
		(7)
		Total 10

Notes:

(a)**B1:** Sets both equations equal and replaces $\sec\left(\theta-\frac{\pi}{3}\right)$ with $\frac{1}{\cos\left(\theta-\frac{\pi}{3}\right)}$

M1: Uses the identity $\cos\theta=\sin\left(\theta+\frac{\pi}{2}\right)$ and simplifies using the identity $\sin 2\theta\equiv 2\sin\theta\cos\theta$ to obtain an expression of the form $\sin 2\theta=k$ then solves an equation of the form $\sin 2\theta=k$ to find a value for θ

A1: r and θ correctly obtained from correct work

ALT

B1: Sets both equations equal and replaces $\sec\left(\theta-\frac{\pi}{3}\right)$ with $-\frac{1}{\cos\left(\theta+\frac{2\pi}{3}\right)}$

M1: Uses the identity $\cos\theta=\sin\left(\theta+\frac{\pi}{2}\right)$ and simplifies using the identity $\sin 2\theta\equiv 2\sin\theta\cos\theta$ to obtain an expression of the form $\sin 2\theta=k$ then solves an equation of the form $\sin 2\theta=k$ to find a value for θ

A1: r and θ correctly obtained from correct work

(b) **Way 1**

M1: Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark)

M1: Correct form for the integration of $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta$ **or** correct form for integration of $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$

M1: Correct form for the integration of both $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(\sec\left(\theta-\frac{\pi}{3}\right)\right)^2 \mathrm{d}\theta$ **and** $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$

NOTE: for $\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta$ they may expand and use identities e.g.

$$\left(\frac{1}{2}\right)\int_{\ldots}^{\ldots}\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2 \mathrm{d}\theta = \int 1+\cos\left(2\theta+\frac{\pi}{3}\right) = \int 1+\cos 2\theta\cos\frac{\pi}{3}-\sin 2\theta\sin\frac{\pi}{3}$$

$$= \theta+\frac{1}{4}\sin 2\theta+\frac{\sqrt{3}}{4}\cos 2\theta$$

There may be variations on the above approach (too many to list on the MS) but if candidates choose this method, then it must be a complete method to score the M mark (use of correct identities condoning sign errors) and M0 if clear differentiation.

A1: For one correct integral (condone the omission of the $\frac{1}{2}$ for this mark)

A1: For both correct integrals (condone the omission of the $\frac{1}{2}$ for this mark)

dM1: Dependent on the first M mark. Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark.

A1: Correct final answer in exact equivalent form

(b) **Way 2**

M1: Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark)

M1: Correct attempt at the area of the small triangle. Condone numerical slips etc, but must see evidence of the half and evidence of using the correct horizontal and vertical distances

A1: Correct exact area obtained

M1: Correct form for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$

A1: Fully correct expression for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ but condone the omission of the $\frac{1}{2}$ for this mark

dM1: Dependent on the first M mark. Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark

A1: Correct final answer in exact equivalent form

(b) **Way 3**

M1: Correct overall strategy mark (may not be scored until the end and condone the omission of the $\frac{1}{2}$'s for this mark)

M1: Correct attempt at the area of the largest triangle. Condone numerical slips etc, but must see evidence of the half and evidence of using the correct horizontal and vertical distances

A1: Correct exact area obtained

M1: Correct form for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$

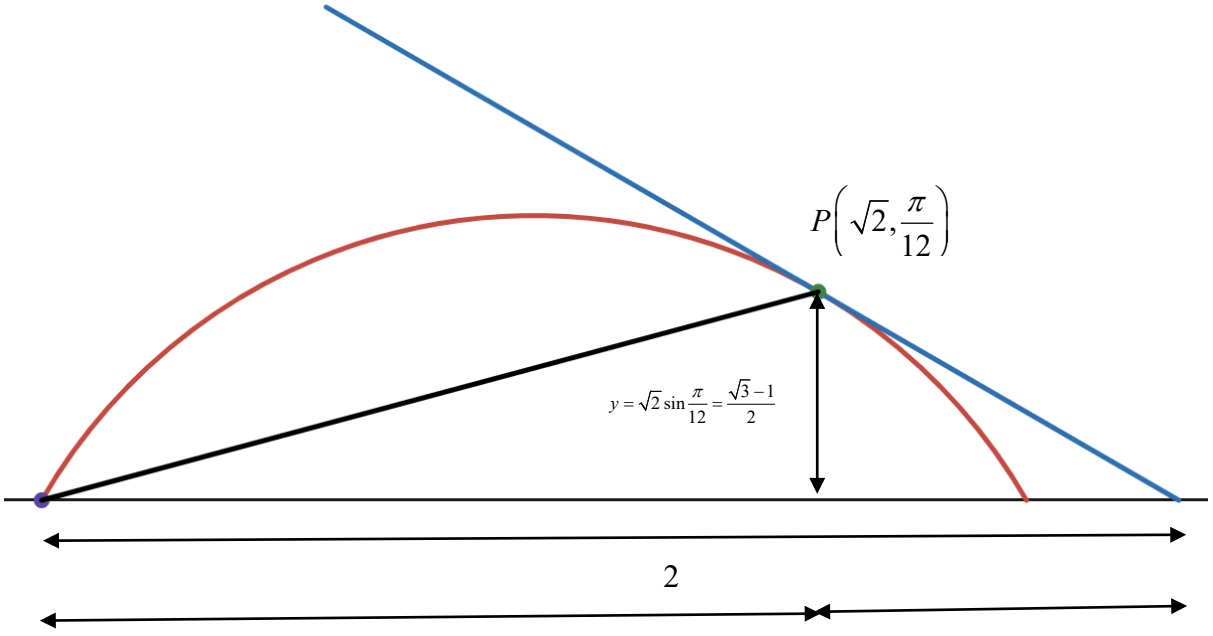
A1: Fully correct expression for the integration of $\left(2\cos\left(\theta+\frac{\pi}{6}\right)\right)^2$ but condone the omission of the $\frac{1}{2}$ for this mark

dM1: Dependent on the first M mark. Applies the correct limits the correct way round and makes progress to obtain an expression for the area. The $\frac{1}{2}$'s must be included for this mark

A1: Correct final answer in exact equivalent form

	TOTAL FOR PAPER: 75 MARKS

Useful diagram for Q8



$$\sqrt{2}\cos\frac{\pi}{12}=\frac{\sqrt{3}+1}{2} \qquad 2-\sqrt{2}\cos\frac{\pi}{12}=\frac{3-\sqrt{3}}{2}$$

$$\text{Area of small } \Delta=\frac{1}{2}\left(\frac{3-\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}-1}{2}\right)=\frac{2\sqrt{3}-3}{4}$$

$$\text{Area of large } \Delta=\frac{1}{2}\left(\frac{\sqrt{3}+1}{2}\right)\left(\frac{\sqrt{3}-1}{2}\right)=\frac{1}{4}$$

$$\text{Area of largest } \Delta=\frac{1}{2}(2)\left(\frac{\sqrt{3}-1}{2}\right)=\frac{\sqrt{3}-1}{2}$$