8.

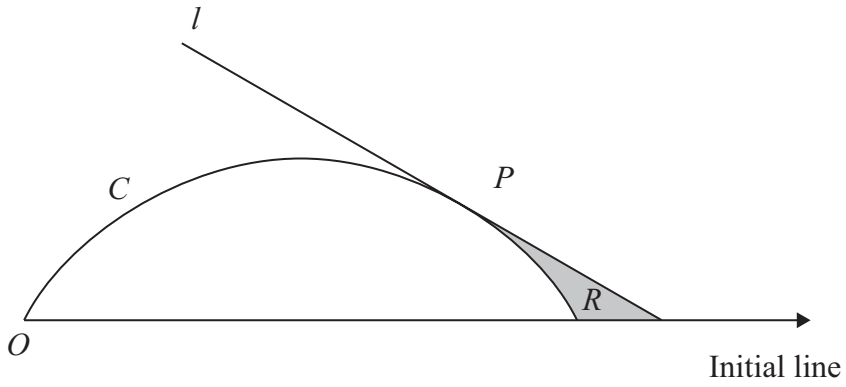


Figure 1

Figure 1 shows a sketch of the curve C with polar equation

$$r = 2\cos\left(\theta + \frac{\pi}{6}\right) \qquad 0 \leqslant \theta \leqslant \frac{\pi}{3}$$

and the line segment l with polar equation

$$r = \sec\left(\theta - \frac{\pi}{3}\right) \qquad 0 \leqslant \theta \leqslant \frac{\pi}{3}$$

The line l is a tangent to C at the point P, as shown in Figure 1.

(a) By solving simultaneous equations and using the identity $\cos\theta \equiv \sin\left(\theta + \frac{\pi}{2}\right)$

show that the polar coordinates of P are $\left(\sqrt{2}, \frac{\pi}{12}\right)$

(3)

The region R, shown shaded in Figure 1, is bounded by C, l and the initial line.

(b) Use algebraic integration to show that the area of R is

$$a\sqrt{3} + b + c\pi$$

where a, b and c are rational numbers to be determined.

$$\left(\text{You may use } \sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4} \text{ and } \cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4} \text{ without justification.}\right)$$

(7)

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS