Question Number	Scheme	Marks
7(a)	$I_n = \int \cosh^n x \, \mathrm{d}x = \int \cosh x \cosh^{n-1} x \, \mathrm{d}x$ $\left(I_n =\right) \int \cosh x \cosh^{n-1} x \, \mathrm{d}x$ seen explicitly or used	B1
	$I_n = \pm \cosh^{n-1} x \sinh x \pm k \int \cosh^{n-2} x \sinh^2 x \, \mathrm{d}x$ Uses parts in the correct direction to obtain an expression of the above form	M1
	$I_n = \cosh^{n-1} x \sinh x - \int (n-1) \cosh^{n-2} x \sinh^2 x \mathrm{d}x$ Correct expression	A1
	$I_n = \cosh^{n-1} x \sinh x - \int (n-1) \cosh^{n-2} x \left(\cosh^2 x - 1\right) \mathrm{d}x$ Use of $\sinh^2 x = \pm \cosh^2 x \pm 1$ **Dependent on the previous method mark**	**d**M1
	$I_n = \cosh^{n-1} x \sinh x - (n-1) I_n + (n-1) I_{n-2}$ Sub for I_n and I_{n-2} and collect terms **Dependent on both previous method marks**	**dd**M1
	$nI_n = \sinh x \cosh^{n-1} x + (n-1) I_{n-2}$ *	A1cso
Do not be overly concerned with notational errors e.g. if a "cosh" becomes a "cos" or a "sinh" becomes a "sin" and is then recovered or if e.g. a $\cosh^2 x$ appears as $\cosh x^2$ but is then recovered or if the odd "x" or "dx" disappears etc. as long as the intention is clear. However, if there are any obvious errors such as sign errors then the final mark should be withheld.		
		(6)
(a) Way 2	$I_n = \int \cosh^n x \, \mathrm{d}x = \int \cosh^2 x \cosh^{n-2} x \, \mathrm{d}x$ $\left(I_n =\right) \int \cosh^2 x \cosh^{n-2} x \, \mathrm{d}x$ seen explicitly or used	B1
	$= \int \left(1 + \sinh^2 x\right) \cosh^{n-2} x \, \mathrm{d}x$ Use of $\cosh^2 x = \pm \sinh^2 x \pm 1$	M1
	$= \int \cosh^{n-2} x \, \mathrm{d}x + \int \sinh x \sinh x \cosh^{n-2} x \, \mathrm{d}x$	
	$= \left(\int \cosh^{n-2} x \, \mathrm{d}x + \right) \sinh x \frac{\cosh^{n-1} x}{n-1} - \int \cosh x \frac{\cosh^{n-1} x}{n-1} \mathrm{d}x$ M1: Integrates $\sinh x \sinh x \cosh^{n-2} x$ to obtain an expression of the form $\pm P \sinh x \cosh^{n-1} x \pm Q \int \cosh x \cosh^{n-1} x \, \mathrm{d}x$ **Dependent on the previous method mark** A1: For $\ldots + \sinh x \frac{\cosh^{n-1} x}{n-1} - \int \cosh x \frac{\cosh^{n-1} x}{n-1} \mathrm{d}x$ Mark in the order on ePEN so that the dM1 is the A1 on ePEN and the A1 is the M2 on ePEN	**d**M1A1 **Note that these 2 marks are reversed for this way.**
	$\Rightarrow (n-1) I_n = (n-1) I_{n-2} + \sinh x \cosh^{n-1} x - I_n$ Sub for I_n and I_{n-2} and collect term **Dependent on both previous method marks**	**dd**M1
	$nI_n = \sinh x \cosh^{n-1} x + (n-1) I_{n-2}$ *	A1cso

(b)	$\int \cosh^4 x\,\mathrm{d}x = \frac{1}{4}\left[\cosh^3 x\sinh x + 3I_2\right]$ or $4\int \cosh^4 x\,\mathrm{d}x = \cosh^3 x\sinh x + 3I_2$ Applies the reduction formula to I_4	M1
	$\int \cosh^4 x\,\mathrm{d}x = \frac{1}{4}\left[\cosh^3 x\sinh x + 3\left(\frac{1}{2}\left[\cosh x\sinh x + I_0\right]\right)\right]$ or $4\int \cosh^4 x\,\mathrm{d}x = \cosh^3 x\sinh x + 3\times\frac{1}{2}\left[\cosh x\sinh x + I_0\right]$ Attempts to use the reduction formula again to obtain I_2 in terms of I_0 May be seen embedded in their I_4 **This is a method mark so allow confusion with the constants.** This mark can also be scored by an attempt to integrate $\cosh^2 x$: Either $\int \cosh^2 x\,\mathrm{d}x = \frac{1}{2}\int \left(\pm\cosh 2x \pm 1\right)\mathrm{d}x = \alpha\sinh 2x + \beta x$ or $\int \cosh^2 x\,\mathrm{d}x = \int \left(\frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2}\right)^2 \mathrm{d}x = \frac{1}{4}\int \left(\mathrm{e}^{2x} + 2 + \mathrm{e}^{-2x}\right)\mathrm{d}x = \frac{1}{8}\mathrm{e}^{2x} + \frac{1}{2}x - \frac{1}{8}\mathrm{e}^{-2x}$	M1
	<u>Note that the final 2 A marks are only to be awarded once I_0 has been evaluated and substituted and they depend on having scored at least one method mark.</u> **Examples:** • $\int \cosh^4 x\,\mathrm{d}x = \frac{1}{4}\cosh^3 x\sinh x + \frac{3}{8}\cosh x\sinh x + \frac{3}{8}x\left(+c\right)$ • $\int \cosh^4 x\,\mathrm{d}x = \frac{1}{4}\cosh^3 x\sinh x + \frac{3}{16}\sinh 2x + \frac{3}{8}x\left(+c\right)$ • $\int \cosh^4 x\,\mathrm{d}x = \frac{1}{4}\cosh^3 x\sinh x + \frac{3}{32}\mathrm{e}^{2x} - \frac{3}{32}\mathrm{e}^{-2x} + \frac{3}{8}x\left(+c\right)$ A1: $\frac{1}{4}\cosh^3 x\sinh x + 1$ other correct term But note that $\frac{3}{32}\mathrm{e}^{2x} - \frac{3}{32}\mathrm{e}^{-2x}$ counts as 1 term A1: Fully correct Correct answer – **constant of integration not required.** **NOTE:** $\frac{1}{4}\left[\cosh^3 x\sinh x + \frac{3}{2}\left(\sinh x\cosh x + I_0\right)\right]$ scores M1M1A0A0 $\frac{1}{4}\left[\cosh^3 x\sinh x + \frac{3}{2}\left(\sinh x\cosh x + x\right)\right]$ scores M1M1A1A1	A1A1
		(4)
		[Total 10]

Note that part (b) can be done in reverse order, in which case the method marks are awarded the other way round e.g.

$$\left(I_0 = \int \mathrm{d}x = x\right)$$

Second M: $I_2 = \frac{1}{2}\left[\sinh x \cosh x + x\right]$ **or as defined in the main scheme**

First M: $I_4 = \frac{1}{4}\left[\sinh x \cosh^3 x + 3\left(\frac{1}{2}\left[\cosh x \sinh x + x\right]\right)\right]$

$$\int \cosh^4 x \,\mathrm{d}x = \frac{1}{4}\cosh^3 x \sinh x + \frac{3}{8}\cosh x \sinh x + \frac{3}{8}x\left(+c\right)$$

A1A1: As defined in the main scheme