Question Number	Scheme	Marks
8(a) Way 1	$\frac{x^2}{a^2}+\frac{(mx+c)^2}{b^2}=1=...$ Eliminates y from the equation of the ellipse and attempts to expand $(mx+c)^2$	M1
	$\frac{x^2}{a^2}+\frac{m^2x^2+2cmx+c^2}{b^2}=1$ Correct equation with the $(mx + c)^2$ expanded correctly	A1
	$4a^4m^2c^2=4a^2\left(b^2+a^2m^2\right)\left(c^2-b^2\right)$ Uses discriminant = 0 or equivalent **Dependent on the first method mark**	**d**M1
	$a^2m^2c^2=b^2c^2-b^4+a^2m^2c^2-a^2m^2\times b^2$ $c^2=b^2+a^2m^2$ * Complete to obtain the GIVEN result with no errors seen. At least one intermediate step should be shown.	A1 cso
		(4)
(a) Way 2	$m=\frac{b\cos\theta}{-a\sin\theta}\Rightarrow y-b\sin\theta=\frac{b\cos\theta}{-a\sin\theta}\left(x-a\cos\theta\right)$ $\Rightarrow y=\frac{b\cos\theta}{-a\sin\theta}x+\frac{b}{\sin\theta}\Rightarrow m=\frac{b\cos\theta}{-a\sin\theta},\ \ c=\frac{b}{\sin\theta}$ M1: Forms the equation of a general tangent and "extracts" c and m A1: Correct c and m	M1A1
	$b^2+a^2m^2=b^2+a^2\left(\frac{b\cos\theta}{-a\sin\theta}\right)^2$ — Substitutes their m into $b^2 + a^2m^2$ or equivalent work. **Dependent on the first method mark**	dM1
	$b^2+a^2m^2=\frac{b^2\cos^2\theta+b^2\sin^2\theta}{\sin^2\theta}=\frac{b^2}{\sin^2\theta}=c^2$ $b^2+a^2m^2=c^2$ * Completes correctly with conclusion e.g. tick, QED etc.	A1

(b) Way 1	$x=0 \Rightarrow y=c, \quad y=0 \Rightarrow x=-\frac{c}{m}$ *or* $(A \text{ is }) \left(-\frac{c}{m},\ 0\right)$ $(B \text{ is }) (0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB = -\frac{c^2}{2m}\left(\text{or } \frac{c^2}{2m}\right)$	Correct expression for the area (allow + or – here)	B1
	$= -\frac{b^2+a^2m^2}{2m}\left(\text{or } \frac{b^2+a^2m^2}{2m}\right)$	Uses $c^2 = b^2 + a^2m^2$ in the area expression to eliminate c.	M1
		Correct expression (may be unsimplified) (allow + or – here)	A1
	$\frac{\mathrm{d}A}{\mathrm{d}m} = \frac{b^2}{2}m^{-2} - \frac{a^2}{2}$ or $\frac{\mathrm{d}A}{\mathrm{d}m} = -\frac{2m \times 2a^2m - 2\left(b^2+a^2m^2\right)}{4m^2}$ Differentiates wrt m (must be correct differentiation for their A). **Dependent on the previous M**		dM1
	At min $m^2 = \frac{b^2}{a^2}$ $m = (\pm)\frac{b}{a}$	Equate their derivative to 0 and solves for m^2 or m. **Dependent on all the previous M's**	ddM1
	Min area $= -\frac{b^2+b^2}{-\frac{2b}{a}} = ab$ $\left(\text{units}^2\right)$	Correct completion with no errors.	A1
			(7)
			[Total 11]

(b) Way 2	$x=0 \Rightarrow y=c, \quad y=0 \Rightarrow x=-\frac{c}{m}$ *or* $(A \text{ is }) \left(-\frac{c}{m},\ 0\right)$ $(B \text{ is }) (0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB = -\frac{c^2}{2m}\left(\text{or } \frac{c^2}{2m}\right)$	Correct expression for the area (allow + or – here)	B1
	$= -\frac{b^2+a^2m^2}{2m}\left(\text{or } \frac{b^2+a^2m^2}{2m}\right)$	Uses $c^2 = b^2 + a^2m^2$ in the area expression to eliminate c.	M1
		Correct expression (may be unsimplified) (allow + or – here)	A1
	$A = -\frac{(am+b)^2 - 2amb}{2m}$	Correct completion of the square in the numerator. **Dependent on the previous M**	dM1
	$= ab - \frac{(am+b)^2}{2m}$ is minimum when $am + b = 0$ Correct argument for establishing the minimum **Dependent on all the previous M's**		ddM1
	Min area $= ab$ $\left(\text{units}^2\right)$	Correct completion with no errors.	A1

(b) Way 3	$x=0 \Rightarrow y=\dfrac{ab\cos^2\theta}{a\sin\theta}+b\sin\theta\left(=\dfrac{b}{\sin\theta}\right)$ **or** $y=0 \Rightarrow y=\dfrac{ab\sin^2\theta}{b\cos\theta}+a\cos\theta\left(=\dfrac{a}{\cos\theta}\right)$ Correct value for one of the intercepts or correct coordinates.		B1
	$x=0 \Rightarrow y=\dfrac{ab\cos^2\theta}{a\sin\theta}+b\sin\theta\left(=\dfrac{b}{\sin\theta}\right)$ **and** $y=0 \Rightarrow y=\dfrac{ab\sin^2\theta}{b\cos\theta}+a\cos\theta\left(=\dfrac{a}{\cos\theta}\right)$ Correct values for both intercepts or correct coordinates.		B1
	Area ΔOAB $A=\dfrac{1}{2}\dfrac{a}{\cos\theta}\dfrac{b}{\sin\theta}$	Correct method for the area using their intercepts	M1
		Correct area (may be unsimplified)	A1
	$\dfrac{\mathrm{d}A}{\mathrm{d}\theta}=\dfrac{-2ab\left(\cos^2\theta-\sin^2\theta\right)}{4\sin^2\theta\cos^2\theta}=0\Rightarrow\theta=\dfrac{\pi}{4}$ Adopts a correct strategy for finding θ at the minimum **Dependent on the previous M**		dM1
	$A_{\min}=\dfrac{ab}{2\sin\dfrac{\pi}{4}\cos\dfrac{\pi}{4}}$	Uses their value for θ to find the minimum value. **Dependent on all the previous M's**	ddM1
	$=ab\ \ \left(\text{units}^2\right)$	Cso	A1
	Alternative for last 3 marks:		
	Area ΔOAB $A=\dfrac{ab}{2\sin\theta\cos\theta}=\dfrac{ab}{\sin 2\theta}$ And the minimum will occur when sin 2θ is maximum i.e. when sin 2θ = 1 Score this mark for a valid argument for determining the minimum **Dependent on the previous M**		dM1
	$A_{\min}=\dfrac{ab}{1}$	Completes the process of finding the minimum. **Dependent on all the previous M's**	ddM1
	$=ab\ \ \left(\text{units}^2\right)$	Correct completion with no errors.	A1

(b) Way 4	$x=0 \Rightarrow y=c, \quad y=0 \Rightarrow x=-\frac{c}{m}$ *or* $(A \text{ is }) \left(-\frac{c}{m},\ 0\right)$ $(B \text{ is }) (0,\ c)$ Correct values for the intercepts or correct coordinates.		B1
	Area $\Delta OAB = -\frac{c^2}{2m}\left(\text{or } \frac{c^2}{2m}\right)$	Correct expression for the area (allow + or – here)	B1
	$= \frac{ac^2}{2\sqrt{c^2-b^2}}\left(= -\frac{ac^2}{2\sqrt{c^2-b^2}}\right)$	Uses $c^2 = b^2 + a^2m^2$ in the area expression to eliminate m.	M1
		Correct expression. (allow + or – here)	A1
	$\frac{\mathrm{d}A}{\mathrm{d}c} = \frac{2ac \times 2\sqrt{c^2-b^2} - 2ac^3\left(c^2-b^2\right)^{-\frac{1}{2}}}{4\left(c^2-b^2\right)}$ Differentiates wrt c. **Dependent on the previous M**		dM1
	At min $2c^2 - 2b^2 - c^2 = 0 \Rightarrow c^2 = 2b^2$ Equate their derivative to 0 and solves to obtain c in terms of b **Dependent on all the previous M's**		dM1
	Min area $= \frac{2ab^2}{2\sqrt{2b^2-b^2}} = ab \ \left(\text{units}^2\right)$	Correct completion with no errors.	A1

There will be other valid methods in part (b). Generally, the first 4 marks are for obtaining an expression for the area of *AOB* and then applying the result in part (a) to enable progress to be made in establishing the minimum or for using the general tangent in terms of θ to find the intercepts and hence the area of *AOB*. The final 3 marks are for selecting and implementing a correct strategy for proving that the minimum area is *ab*.

There may also be other valid methods in part (a).

If you are in any doubt whether a particular method is valid then please seek advice from your Team Leader.

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom