Question Number	Scheme	Notes	Marks
8(a)	$\mathbf{n}=\begin{pmatrix}1\\-5\\3\end{pmatrix}\times\begin{pmatrix}3\\-2\\2\end{pmatrix}=\begin{pmatrix}-10+6\\-(2-9)\\-2+15\end{pmatrix}$	Attempt vector product between normal vectors	M1
	$=\begin{pmatrix}-4\\7\\13\end{pmatrix}$	Correct vector	A1
	$x=0\Rightarrow -5y+3z=11,\quad -2y+2z=7$ $\Rightarrow y=-\frac{1}{4},z=\frac{13}{4}$ or $y=0\Rightarrow x+3z=11,\quad 3x+2z=7$ $\Rightarrow x=-\frac{1}{7},z=\frac{26}{7}$ or $z=0\Rightarrow x-5y=11,3x-2y=7$ $\Rightarrow x=1,y=-2$	Correct strategy to find a point on *l*	M1
		Correct position vector of point on *l*	A1
	$\mathbf{r}=\mathbf{i}-2\mathbf{j}+\lambda(-4\mathbf{i}+7\mathbf{j}+13\mathbf{k})$	Correct equation. (follow through their position and direction vectors but must be "**r** = ")	A1ft
			(5)
ALT	$x=11+5y-3z$		
	$3x-2y+2z=7\Rightarrow 3(11+5y-3z)-2y+2z=7$ $\Rightarrow y-\frac{7z}{13}=-\frac{26}{13}\quad\left(z=\frac{13y+26}{7}\right)$ Eliminate one variable		M1
	$x=11+5\left(-\frac{26}{13}+\frac{7z}{13}\right)\Rightarrow z=\frac{13-13x}{4}$	Obtain 2 correct expressions for one of the variables	A1
	$\frac{x-1}{-\frac{4}{13}}=\frac{y+2}{\frac{7}{13}}=z$	M1 Obtain a Cartesian equation for *l* A1 Correct equation	M1A1
	$\mathbf{r}=(\mathbf{i}-2\mathbf{j})+\lambda\left(-\frac{4}{13}\mathbf{i}+\frac{7}{13}\mathbf{j}+\mathbf{k}\right)$ oe	Deduce a vector equation for *l* Follow through their Cartesian equation	A1ft
			(5)

(b)	$\begin{pmatrix}3\\2\\1\end{pmatrix}-\begin{pmatrix}2\\0\\3\end{pmatrix}=\begin{pmatrix}1\\2\\-2\end{pmatrix}$	Correct vector joining P to Q	B1
	$\begin{pmatrix}-4\\7\\13\end{pmatrix}\times\begin{pmatrix}1\\2\\-2\end{pmatrix}=\begin{pmatrix}-40\\5\\-15\end{pmatrix}$	Attempt vector product between the direction of l and their $\mathbf{i}+2\mathbf{j}-2\mathbf{k}$	M1
		Correct vector	A1
	$\sin\theta=\dfrac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert\lvert\mathbf{i}+2\mathbf{j}-2\mathbf{k}\rvert}$	Angle between PQ and line n	
	$d=\lvert\overrightarrow{PQ}\rvert\sin\theta$		
	$d=\dfrac{\lvert-40\mathbf{i}+5\mathbf{j}-15\mathbf{k}\rvert}{\lvert-4\mathbf{i}+7\mathbf{j}+13\mathbf{k}\rvert}=\dfrac{1}{\sqrt{234}}\sqrt{40^2+5^2+15^2}$	Fully correct method for the distance	M1
	$d=\dfrac{5\sqrt{481}}{39}$	Cao Allow equivalent **exact** forms e.g. $d=\dfrac{5\sqrt{74}}{\sqrt{234}}$	A1
			(5)
ALT 1	$\mathbf{r}_m=\begin{pmatrix}2\\0\\3\end{pmatrix}+\lambda\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$ or $\mathbf{r}_n=\begin{pmatrix}3\\2\\1\end{pmatrix}+\mu\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}$	Vector equation for either line with their direction vector from (a)	B1ft
	$\overrightarrow{OP}=\begin{pmatrix}2\\0\\3\end{pmatrix}$ $\overrightarrow{ON}=\begin{pmatrix}3-\frac{4}{7}\mu\\2+\mu\\1+\frac{13}{7}\mu\end{pmatrix}$ $\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}$	Uses either P and the parametric form of a point on n OR Q and the parametric form of a point on m	
	$\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}.\begin{pmatrix}-\frac{4}{7}\\1\\\frac{13}{7}\end{pmatrix}=0$	M1: Forms scalar product of vector NP and direction vector of l and equates to zero A1: Correct vectors	M1A1
	$\Rightarrow\mu=\dfrac{56}{117}$	Solves	M1
	$\Rightarrow d=\sqrt{\left(-\frac{85}{117}\right)^2+\left(-\frac{290}{117}\right)^2+\left(\frac{10}{9}\right)^2}=\dfrac{5\sqrt{481}}{39}$	Obtains the correct distance	A1
			(5)
	Alternative for M1A1M1		
	$\overrightarrow{NP}=\begin{pmatrix}-1+\frac{4}{7}\mu\\-2-\mu\\2-\frac{13}{7}\mu\end{pmatrix}\Rightarrow d=\sqrt{\left(-1+\frac{4}{7}\mu\right)^2+(-2-\mu)^2+\left(2-\frac{13}{7}\mu\right)^2}\Rightarrow d$ is min when $\Rightarrow\mu=\dfrac{56}{117}$ M1: Find d in terms of a parameter A1: correct expression M1: use calculus (or simplify and complete the square) to find the parameter corresponding to the min d		

ALT 2	Correct vector PQ		B1
	$\begin{pmatrix}1\\2\\-2\end{pmatrix}.\begin{pmatrix}-4\\7\\13\end{pmatrix}=\begin{vmatrix}1\\2\\-2\end{vmatrix}\begin{vmatrix}-4\\7\\13\end{vmatrix}\cos\theta$	Forms the scalar product and attempts to evaluate the LHS	M1
	$\cos\theta=\dfrac{-16}{3\sqrt{234}}$	Correct value for $\cos\theta$ exact or decimal	A1
	$d=\lvert PQ\rvert\sin\theta=3\sqrt{1-\left(\dfrac{-16}{3\sqrt{234}}\right)^2}=\dfrac{5\sqrt{74}}{\sqrt{234}}$	M1: Correct method for the distance. A1: Correct **EXACT** distance	M1A1
			(5)
			Total 10

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