Question Number	Scheme	Notes	Marks
1(a)	$1-\tanh^2 x \equiv \operatorname{sech}^2 x$		
	$1-\tanh^2 x = 1-\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)^2$	Replaces the tanh x on the lhs with a **<u>correct</u>** expression in terms of exponentials.	B1
	$=\dfrac{(e^x+e^{-x})^2-(e^x-e^{-x})^2}{(e^x+e^{-x})^2}=\dfrac{(e^{2x}+2+e^{-2x})-(e^{2x}-2+e^{-2x})}{(e^x+e^{-x})^2}$ or e.g. $\dfrac{2e^{2x}\times 2e^{-2x}}{(e^x+e^{-x})^2}$ Attempts to find common denominator and expand numerator		M1
	$=\left(\dfrac{4}{(e^x+e^{-x})^2}\right)=\operatorname{sech}^2 x*$	Obtains the rhs with no errors.	A1cso
			(3)
ALT 1	$1-\tanh^2 x=(1-\tanh x)(1+\tanh x)$ $=\left(1-\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)\right)\left(1+\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)\right)$	Uses the difference of 2 squares on the lhs and replaces the tanh x with a **<u>correct</u>** expression in terms of exponentials.	B1
	$=\left(\dfrac{2e^{-x}}{e^x+e^{-x}}\right)\left(\dfrac{2e^x}{e^x+e^{-x}}\right)$	Attempt to find common denominators and simplify numerators.	M1
	$=\left(\dfrac{4}{(e^x+e^{-x})^2}\right)=\operatorname{sech}^2 x*$	Obtains the rhs with no errors.	A1cso
ALT 2	$\operatorname{sech}^2 x=\dfrac{4}{(e^x+e^{-x})^2}$	Replaces the sech x on the rhs with a **<u>correct</u>** expression in terms of exponentials.	B1
	$=\dfrac{(e^{2x}+2+e^{-2x})-(e^{2x}-2+e^{-2x})}{(e^x+e^{-x})^2}=\dfrac{(e^x+e^{-x})^2-(e^x-e^{-x})^2}{(e^x+e^{-x})^2}$ Attempts to express the "4" in terms of the denominator.		M1
	$=1-\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)^2=1-\tanh^2 x*$	Obtains the lhs with no errors.	A1cso

(b)	$2\operatorname{sech}^2 x + 3\tanh x = 3 \Rightarrow 2(1-\tanh^2 x) + 3\tanh x = 3$ $\Rightarrow 2\tanh^2 x - 3\tanh x + 1 = 0$ Uses $\operatorname{sech}^2 x = 1 - \tanh^2 x$ and forms a 3 term quadratic in $\tanh x$		M1
	$(2\tanh x - 1)(\tanh x - 1) = 0 \Rightarrow \tanh x = ...$	Solves 3TQ by any valid method including calculator.	M1
	$\tanh x = \frac{1}{2} \rightarrow x = \ln\sqrt{3}$	$\ln\sqrt{3}$. Accept $\frac{1}{2}\ln 3$, $-\frac{1}{2}\ln\frac{1}{3}$ And no other answers.	A1
			(3)
ALT	$2\operatorname{sech}^2 x + 3\tanh x = 3 \Rightarrow 2\left(\frac{4}{(e^x + e^{-x})^2}\right) + 3\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = 3$ $\Rightarrow 8 + 3(e^{2x} - e^{-2x}) = 3(e^{2x} + 2 + e^{-2x}) \Rightarrow ...$ Substitutes the correct exponential forms, attempts to eliminate fractions and collect terms		M1
	$6e^{-2x} = 2 \Rightarrow e^{-2x} = \frac{1}{3}$	Rearranges to reach $e^{-2x} = ...$	M1
	$x = \ln\sqrt{3}$	$\ln\sqrt{3}$. Accept $\frac{1}{2}\ln 3$, $-\frac{1}{2}\ln\frac{1}{3}$ And no other answers.	A1
			Total 6