5(a)	$\int \sec^n x\,\mathrm{d}x = \int \sec^{n-2} x \sec^2 x \mathrm{d}x$	Splits $\sec^n x$ into $\sec^{n-2} x \sec^2 x$	M1
	$\int \sec^n x\,\mathrm{d}x = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x \tan^2 x\,\mathrm{d}x$ **Depends on previous M mark** dM1: Uses integration by parts to obtain $\sec^{n-2} x \tan x - k\int \sec^{n-2} x \tan^2 x\,\mathrm{d}x$ A1: Correct integration		**d**M1A1
	$\int \sec^n x\,\mathrm{d}x = \sec^{n-2} x \tan x - \int (n-2)\sec^{n-2} x(\sec^2 x - 1)\,\mathrm{d}x$ Uses $\tan^2 x = \sec^2 x - 1$		B1 **(M1 on EPEN)**
	$\int \sec^n x\,\mathrm{d}x = \sec^{n-2} x \tan x - (n-2)\int \sec^n x\,\mathrm{d}x + (n-2)\int \sec^{n-2} x\,\mathrm{d}x$ $= \sec^{n-2} x \tan x - (n-2)I_n + (n-2)I_{n-2} \Rightarrow (n-1)I_n = \ldots$ **Depends on all previous M and B marks** Introduces I_n and I_{n-2} and makes progress to the given result.		**dd**M1
	$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ *	Fully correct proof.	A1cso
			(6)
ALT	$\int \sec^n x\,\mathrm{d}x = \int \sec^{n-2} x \sec^2 x \mathrm{d}x$	Splits $\sec^n x$ into $\sec^{n-2} x \sec^2 x$	M1
	$\int \sec^{n-2} x \sec^2 x \mathrm{d}x = \int \sec^{n-2} x (1 + \tan^2 x)\mathrm{d}x$ $= \int \sec^{n-2} x \mathrm{d}x + \int \tan^2 x \sec^{n-2} x \mathrm{d}x$	Uses $\sec^2 x = 1 + \tan^2 x$ and splits into 2 integrals.	B1 **(4th mark M1 on EPEN)**
	$\int \tan^2 x \sec^{n-2} x \mathrm{d}x = \frac{1}{(n-2)} \tan x \sec^{n-2} x - \frac{1}{(n-2)}\int \sec^n x\,\mathrm{d}x$ Uses integration by parts on $\int \tan^2 x \sec^{n-2} x \mathrm{d}x$ to obtain $A \tan x \sec^{n-2} x - B\int \sec^n x\,\mathrm{d}x$ **Note this is the 2nd M on EPEN.**		**d**M1
	$\int \sec^n x\,\mathrm{d}x = \int \sec^{n-2} x\,\mathrm{d}x + \frac{1}{(n-2)} \tan x \sec^{n-2} x - \frac{1}{(n-2)}\int \sec^n x\,\mathrm{d}x$ Fully correct integration		A1
	$\int \sec^n x\,\mathrm{d}x = I_{n-2} + \frac{1}{(n-2)} \tan x \sec^{n-2} x - \frac{1}{(n-2)} I_n \Rightarrow (n-1)I_n = \ldots$ **Depends on previous M and B marks** Introduces I_n and I_{n-2} and makes progress to the given result.		**dd**M1
	$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ *	Fully correct proof.	A1cso

5(b)	$I_2 = 1$	Correct value for I_2 seen or implied.	B1
	$I_6 = \frac{1}{5} \tan x \sec^4 x + \frac{4}{5} I_4$ or e.g.	Applies the given reduction formula once.	M1

	$I_6 = \frac{1}{5}\tan\frac{\pi}{4}\sec^4\frac{\pi}{4} + \frac{4}{5}I_4$ or e.g. $I_6 = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{5}I_4$		
	$= \frac{1}{5}\tan x\sec^4 x + \frac{4}{5}\left(\frac{1}{3}\tan x\sec^2 x + \frac{2}{3}I_2\right) = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{15}(1)\left(\sqrt{2}\right)^2 + \frac{8}{15}(1)$ Applies the given reduction formula again and uses the limits to reach a numerical expression for I_6		M1
	$= \frac{28}{15}$	Correct value	A1
			(4)
ALT	$I_2 = 1$	Correct value for I_2 seen or implied.	B1
	$I_4 = \frac{1}{3}\tan x\sec^2 x + \frac{2}{3}I_2$ or e.g. $I_4 = \frac{1}{3}\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} + \frac{2}{3}I_2$ or e.g. $I_4 = \frac{1}{3}(1)\left(\sqrt{2}\right)^2 + \frac{2}{3}I_2$	Applies the given reduction formula once.	M1
	$I_6 = \frac{1}{5}\tan x\sec^4 x + \frac{4}{5}\left(\frac{1}{3}\tan x\sec^2 x + \frac{2}{3}I_2\right) = \frac{1}{5}(1)\left(\sqrt{2}\right)^4 + \frac{4}{15}(1)\left(\sqrt{2}\right)^2 + \frac{8}{15}$ Applies the given reduction formula again and uses the limits to reach a numerical expression for I_6		M1
	$= \frac{28}{15}$	Correct value	A1
			Total 10

In part (b), condone confusion with the coefficients provided the intention is clear.

For either method in part (b), all working must be shown and the given reduction formula must be used at least once. So do not allow e.g. I_4 to be evaluated with a calculator but I_4 can be evaluated directly without using the given reduction formula using an alternative method e.g. by parts or by substitution – see below:

Parts:

$$I_4 = \int \sec^4 x\,\mathrm{d}x = \int \sec^2 x\sec^2 x\,\mathrm{d}x = \sec^2 x\tan x - 2\int \sec^2 x\tan^2 x\,\mathrm{d}x$$

$$= \sec^2 x\tan x - 2\int \sec^2 x\left(\sec^2 x - 1\right)\mathrm{d}x = \sec^2 x\tan x - 2\int \sec^4 x\,\mathrm{d}x + 2\int \sec^2 x\,\mathrm{d}x$$

$$= \sec^2 x\tan x - 2I_4 + 2\int \sec^2 x\,\mathrm{d}x \Rightarrow 3I_4 = \sec^2 x\tan x + 2\tan x \Rightarrow I_4 = \frac{1}{3}\sec^2 x\tan x + \frac{2}{3}\tan x$$

Substitution:

$$I_4 = \int \sec^4 x\,\mathrm{d}x = \int \sec^2 x\sec^2 x\,\mathrm{d}x = \int \sec^2 x\left(1+\tan^2 x\right)\mathrm{d}x$$

$$u = \tan x \Rightarrow \int \sec^2 x\left(1+\tan^2 x\right)\mathrm{d}x = \int \sec^2 x\left(1+u^2\right)\frac{\mathrm{d}u}{\sec^2 x} = \frac{u^3}{3} + u = \frac{\tan^3 x}{3} + \tan x$$

Question Number	Scheme	Notes	Marks