Question Number	Scheme	Notes	Marks
7(i)	$x^2-4x+5=(x-2)^2+1$	Attempts to complete the square. Allow for $(x-2)^2+c,\ c>0$	M1
	$\int\frac{1}{(x-2)^2+1}\mathrm{d}x=\arctan(x-2)$	Allow for $k\arctan\mathrm{f}(x)$.	M1
	$\left[\arctan(x-2)\right]_1^2=0-\left(-\frac{\pi}{4}\right)=\frac{\pi}{4}$	$\frac{\pi}{4}$ cao	A1
			(3)
7(ii)	$\int\frac{\sqrt{x^2-3}}{x^2}\mathrm{d}x=-\frac{\sqrt{x^2-3}}{x}+\int\frac{1}{\sqrt{x^2-3}}\mathrm{d}x$ Uses integration by parts and obtains $A\frac{\sqrt{x^2-3}}{x}+B\int\frac{1}{\sqrt{x^2-3}}\mathrm{d}x$		M1
	$=-\frac{\sqrt{x^2-3}}{x}+\operatorname{arcosh}\frac{x}{\sqrt{3}}$	$B\int\frac{1}{\sqrt{x^2-3}}\mathrm{d}x=k\operatorname{arcosh}\mathrm{f}(x)$	M1
		All correct	A1
	$\int_{\sqrt{3}}^{3}\frac{\sqrt{x^2-3}}{x^2}\mathrm{d}x=\left[-\frac{\sqrt{x^2-3}}{x}+\operatorname{arcosh}\frac{x}{\sqrt{3}}\right]_{\sqrt{3}}^{3}=\left(-\frac{\sqrt{6}}{3}+\operatorname{arcosh}\sqrt{3}\right)-(0+\operatorname{arcosh}1)$ Applies the limits 3 and $\sqrt{3}$ **Depends on both previous M marks**		**d**M1
	$\operatorname{arcosh}\sqrt{3}-\frac{1}{3}\sqrt{6}=\ln(\sqrt{2}+\sqrt{3})-\frac{1}{3}\sqrt{6}$	Accept either of these forms.	A1
			(5)
7(ii) ALT 1	$\int\frac{\sqrt{x^2-3}}{x^2}\mathrm{d}x=\int\frac{\sqrt{3\cosh^2u-3}}{3\cosh^2u}\sqrt{3}\sinh u\,\mathrm{d}u$	A complete substitution using $x=\sqrt{3}\cosh u$	M1
	$=\int\tanh^2u\,\mathrm{d}u$	Obtains $k\int\tanh^2u\,\mathrm{d}u$	M1
	$=\int(1-\operatorname{sech}^2u)\mathrm{d}u=u-\tanh u$	Correct integration	A1
	$\int_{\sqrt{3}}^{3}\frac{\sqrt{x^2-3}}{x^2}\mathrm{d}x=\left[u-\tanh u\right]_0^{\operatorname{arcosh}\sqrt{3}}=\operatorname{arcosh}\sqrt{3}-\tanh\left(\operatorname{arcosh}\sqrt{3}\right)-0$ Applies the limits 0 and $\operatorname{arcosh}\sqrt{3}$ **Depends on both previous M marks**		**d**M1
	$\operatorname{arcosh}\sqrt{3}-\frac{1}{3}\sqrt{6}=\ln(\sqrt{2}+\sqrt{3})-\frac{1}{3}\sqrt{6}$	Accept either of these forms.	A1

7(ii) ALT 2	$\int \frac{\sqrt{x^2-3}}{x^2}\,dx = \int \frac{\sqrt{3\sec^2 u - 3}}{3\sec^2 u}\sqrt{3}\sec u \tan u\,du$	A complete substitution using $x = \sqrt{3}\sec u$	M1
	$= \int \frac{\tan^2 u}{\sec u}\,du$	Obtains $k\int \frac{\tan^2 u}{\sec u}\,du$	M1
	$= \ln(\sec u + \tan u) - \sin u$	Correct integration	A1
	$\int_{\sqrt{3}}^{3} \frac{\sqrt{x^2-3}}{x^2}\,dx = \left[\ln(\sec u + \tan u) - \sin u\right]_0^{\text{arcsec}\sqrt{3}}$ $= \ln\left(\sec\left(\text{arcsec}\sqrt{3}\right) + \tan\left(\text{arcsec}\sqrt{3}\right)\right) - \ln\left(\sec(0) + \tan(0)\right) - \sin\left(\text{arcsec}\sqrt{3}\right)$ Applies the limits 0 and arcsec$\sqrt{3}$ **Depends on both previous M marks**		**d**M1
	$\int_{\sqrt{3}}^{3} \frac{\sqrt{x^2-3}}{x^2}\,dx = \ln(\sqrt{2}+\sqrt{3}) - \frac{1}{3}\sqrt{6}$	Correct answer.	A1
			Total 8

Note that there may be other ways to perform the integration in part (ii) e.g. subsequent substitutions. Marks can be awarded if the method leads to something that is integrable and should be awarded as in the main scheme e.g. M1 for a complete method, M2 for simplifying and reaching an expression that itself can be integrated or can be integrated after rearrangement, A1 for correct integration, dM3 for using appropriate limits and A2 as above.

Alternative approach:

$$\int \frac{\sqrt{x^2-3}}{x^2}\,dx = \int \frac{x^2-3}{x^2\sqrt{x^2-3}}\,dx = \int \frac{1}{\sqrt{x^2-3}}\,dx - \int \frac{3}{x^2\sqrt{x^2-3}}\,dx = \text{arcosh}\,\frac{x}{\sqrt{3}} - \ldots$$

Can score **M0M1A0dM0A0** if there is no creditable attempt at the second integral.

If the second integral is attempted, it must be using a suitable method
e.g. with either $x = \sqrt{3}\cosh u$ or $x = \sqrt{3}\sec u$:

$$\int \frac{3}{x^2\sqrt{x^2-3}}\,dx = \int \frac{3}{3\cosh^2 u\sqrt{3\cosh^2 u - 3}}\sqrt{3}\sinh u\,du = \int \text{sech}^2 u\;du = \tanh u + c$$

or

$$\int \frac{3}{x^2\sqrt{x^2-3}}\,dx = \int \frac{3}{3\sec^2 u\sqrt{3\sec^2 u - 3}}\sqrt{3}\sec u\tan u\,du = \int \cos u\;du = \sin u + c$$

In these cases the first M can then be awarded and the other marks as defined with the appropriate limits used.