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8. The hyperbola H has equation

$$4x^2 - y^2 = 4$$

(a) Write down the equations of the asymptotes of H.

(1)

(b) Find the coordinates of the foci of H.

(2)

The point $P(\sec\theta, 2\tan\theta)$ lies on H.

(c) Using calculus, show that the equation of the tangent to H at the point P is

$$y\tan\theta = 2x\sec\theta - 2$$

(4)

The point $V(-1, 0)$ and the point $W(1, 0)$ both lie on H.

The point $Q(\sec\theta, -2\tan\theta)$ also lies on H.

Given that P, Q, V and W are distinct points on H and that the lines VP and WQ intersect at the point S,

(d) show that, as θ varies, S lies on an ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are integers to be found.

(7)

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