Question Number	Scheme	Notes	Marks
1(a)	$8\cosh^4 x = 8\left(\dfrac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}\right)^4 = \dfrac{8}{16}\left(\mathrm{e}^{4x}+4\mathrm{e}^{2x}+6+4\mathrm{e}^{-2x}+\mathrm{e}^{-4x}\right)$ Applies $\cosh x = \dfrac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}$ and attempts to expand the bracket to at least 4 different and no more than 5 different terms of the correct form but they may be "uncollected" depending on how they do the expansion. Allow unsimplified terms e.g. $\left(\mathrm{e}^x\right)^3\mathrm{e}^{-x}$. May see $8\left(\dfrac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}\right)^2\left(\dfrac{\mathrm{e}^x+\mathrm{e}^{-x}}{2}\right)^2$ but must attempt to expand as above		M1
	$=\dfrac{1}{2}\left(\mathrm{e}^{4x}+\mathrm{e}^{-4x}\right)+4\left(\dfrac{\mathrm{e}^{2x}+\mathrm{e}^{-2x}}{2}\right)+3=\ldots$	Collects appropriate terms and reaches the form $\cosh 4x + p\cosh 2x + q$ or obtains values of p and q.	M1
	$=\cosh 4x + 4\cosh 2x + 3$	Correct expression or values e.g. $p = 4$ and $q = 3$	A1
	No marks are available in (a) if exponentials are not used but note that they may appear in combination with the use of hyperbolic identities e.g.: $8\cosh^4 x = 8\left(\cosh^2 x\right)^2 = 8\left(\dfrac{\cosh 2x+1}{2}\right)^2 = 2\left(\dfrac{\mathrm{e}^{2x}+\mathrm{e}^{-2x}}{2}+1\right)^2$ $= 2\left(\dfrac{\mathrm{e}^{4x}+2+\mathrm{e}^{-4x}}{4}+\mathrm{e}^{2x}+\mathrm{e}^{-2x}+1\right) = \dfrac{\mathrm{e}^{4x}+\mathrm{e}^{-4x}}{2}+4\left(\dfrac{\mathrm{e}^{2x}+\mathrm{e}^{-2x}}{2}\right)+2$ $=\cosh 4x + 4\cosh 2x + 3$ **Allow to "meet in the middle" e.g. expands as above and compares with** $\dfrac{1}{2}\left(\mathrm{e}^{4x}+\mathrm{e}^{-4x}\right)+p\left(\dfrac{\mathrm{e}^{2x}+\mathrm{e}^{-2x}}{2}\right)+q \Rightarrow p=\ldots, q=\ldots$ **but to score any marks the expansion must be attempted.**		
			(3)

(b) **Way 1**	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 8\cosh^4 x - 4\cosh 2x - 3 - 17\cosh 2x + 9 = 0$ $\Rightarrow 8\cosh^4 x - 21\cosh 2x + 6 = 0 \Rightarrow 8\cosh^4 x - 21\left(2\cosh^2 x - 1\right) + 6 = 0$ Uses **their** result from part (a) and $\cosh 2x = \pm 2\cosh^2 x \pm 1$ to obtain a quadratic equation in $\cosh^2 x$ **or** $\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\left(2\cosh^2 x - 1\right)^2 - 1 - 17\left(2\cosh^2 x - 1\right) + 9 = 0$ Uses $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ and $\cosh 2x = \pm 2\cosh^2 x \pm 1$ to obtain a quadratic equation in $\cosh^2 x$		M1
	$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$	Correct 3TQ in $\cosh^2 x$	A1
	$\Rightarrow 8\cosh^4 x - 42\cosh^2 x + 27 = 0$ $\Rightarrow \cosh^2 x = \frac{9}{2}\left(, \frac{3}{4}\right)$	Solves 3TQ in $\cosh^2 x$ (apply usual rules if necessary) to obtain $\cosh^2 x = k \ \left(k \in \mathbb{R} \text{ and } > 1\right)$. May be implied by their values – check if necessary.	M1
	$\cosh^2 x = \frac{9}{2} \Rightarrow \cosh x = \frac{3}{\sqrt{2}} \Rightarrow x = \pm \ln\left(\frac{3}{\sqrt{2}} + \sqrt{\frac{9}{2} - 1}\right)$ or $\cosh x = \frac{3}{\sqrt{2}} \Rightarrow \frac{e^x + e^{-x}}{2} = \frac{3}{\sqrt{2}} \Rightarrow \sqrt{2}e^{2x} - 6e^x + \sqrt{2} = 0 \Rightarrow e^x = ... \Rightarrow x = ...$ or $\cosh^2 x = \frac{9}{2} \Rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{9}{2} \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = ... \Rightarrow x = ...$ Takes square root to obtain $\cosh x = k \ \left(k > 1\right)$ and applies the correct logarithmic form for arcosh or uses the correct exponential form for $\cosh x$ to obtain at least one value for x The root(s) must be real to score this mark.		M1
	$x = \pm \ln\left(\frac{3\sqrt{2}}{2} + \frac{\sqrt{14}}{2}\right)$ Both correct and exact including brackets. Accept simplified equivalents e.g. $x = \ln\left(\frac{3}{\sqrt{2}} \pm \frac{\sqrt{7}}{\sqrt{2}}\right)$ but withhold this mark if additional answers are given unless they are the same e.g. allow $x = \pm \ln\left(\frac{3\sqrt{2}}{2} \pm \frac{\sqrt{14}}{2}\right)$		A1
			(5)

(b) Way 2	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow 2\cosh^2 2x - 1 - 17\cosh 2x + 9 = 0$ Applies $\cosh 4x = \pm 2\cosh^2 2x \pm 1$ to obtain a quadratic equation in cosh 2*x*		M1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$	Correct 3TQ in cosh 2*x*	A1
	$2\cosh^2 2x - 17\cosh 2x + 8 = 0$ $\Rightarrow \cosh 2x = 8\left(, \frac{1}{2}\right)$	Solves 3TQ in cosh 2*x* (apply usual rules if necessary) to obtain $\cosh 2x = k \ \ (k \in \mathbb{R} \text{ and } > 1)$	M1
	$\cosh 2x = 8 \Rightarrow 2x = \pm \ln\left(8 + \sqrt{8^2 - 1}\right)$ or $\cosh 2x = 8 \Rightarrow \frac{e^{2x} + e^{-2x}}{2} = 8 \Rightarrow e^{4x} - 16e^{2x} + 1 = 0 \Rightarrow e^{2x} = \ldots \Rightarrow 2x = \ldots$ Applies the correct logarithmic form for arcosh from $\cosh 2x = k \ \ (k > 1)$ or uses the correct exponential form for cosh 2*x* to obtain at least one value for 2*x* The root(s) must be real to score this mark.		M1
	$x = \pm\frac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ or e.g. $x = \pm\ln\left(8 + 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \frac{1}{2}\ln\left(8 \pm \sqrt{63}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
(b) Way 3	$\cosh 4x - 17\cosh 2x + 9 = 0 \Rightarrow \frac{e^{4x} + e^{-4x}}{2} - \frac{17}{2}\left(e^{2x} + e^{-2x}\right) + 9 = 0$ $\Rightarrow e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ M1: Applies the correct exponential forms and attempts a quartic equation in e^{2x} A1: Correct equation		M1A1
	$e^{8x} - 17e^{6x} + 18e^{4x} - 17e^{2x} + 1 = 0$ $\Rightarrow e^{2x} = 8 \pm 3\sqrt{7}, \ldots$	Solves and proceeds to a value for e^{2x} where $e^{2x} > 1$ and real.	M1
	$\Rightarrow e^{2x} = 8 \pm 3\sqrt{7} \Rightarrow 2x = \ln\left(8 \pm 3\sqrt{7}\right)$	Takes ln's to obtain at least one value for 2*x* The root(s) must be real to score this mark.	M1
	$x = \frac{1}{2}\ln\left(8 \pm 3\sqrt{7}\right)$ or e.g. $x = \ln\left(8 \pm 3\sqrt{7}\right)^{\frac{1}{2}}$	Both correct and exact with brackets. Accept simplified equivalents e.g. $x = \pm\frac{1}{2}\ln\left(8 + 3\sqrt{7}\right)$ but withhold this mark if additional answers are given unless they are the same as above.	A1
			Total 8