Question Number	Scheme	Notes	Marks
3(a)	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech } y = \frac{x}{2}$ $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = -2\,\text{sech } y \tanh y$	Takes "sech" of both sides and differentiates to obtain $\frac{\mathrm{d}x}{\mathrm{d}y} = k\,\text{sech } y \tanh y$ or equivalent.	M1
	$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = -2\left(\frac{x}{2}\right)\sqrt{1-\left(\frac{x}{2}\right)^2}$ M1: Replaces sech y with $\frac{x}{2}$ and tanh y with $\sqrt{1-\left(\frac{x}{2}\right)^2}$ A1: Correct equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
			(4)
(a) Way 2	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech } y = \frac{x}{2}$ $\Rightarrow \cosh y = \frac{2}{x} \Rightarrow \sinh y \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{x^2}$	Takes "sech" of both sides, changes to "cosh" and differentiates to obtain $\sinh y \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{x^2}$ or equivalent.	M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{x^2 \sinh y} = -\frac{2}{x^2\sqrt{\left(\frac{2}{x}\right)^2-1}}$ M1: Replaces sinh y with $\sqrt{\left(\frac{2}{x}\right)^2-1}$ A1: Correct equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
(a) Way 3	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow y = \text{arcosh}\left(\frac{2}{x}\right)$ Changes to "arcosh" correctly. **Score this as the second M mark on EPEN.**		M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{\left(\frac{2}{x}\right)^2-1}} \times -\frac{2}{x^2}$ M1: Differentiates to the form $\frac{k}{x^2\sqrt{\left(\frac{2}{x}\right)^2-1}}$ oe A1: Correct equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form in terms of x only. **Score this as the first M mark and first A mark on EPEN.**		M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1

(a) **Way 4**	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech}\, y = \frac{x}{2} \Rightarrow \left(\frac{x}{2}\right)^2 = \text{sech}^2 y \Rightarrow \tanh y = \sqrt{1-\left(\frac{x}{2}\right)^2}$ $\Rightarrow \text{sech}^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = -x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$ Differentiates to $\text{sech}^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = kx\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$ or equivalent		M1
	$\Rightarrow \text{sech}^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = -x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}} \Rightarrow \frac{x^2}{4}\frac{\mathrm{d}y}{\mathrm{d}x} = -x\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4}{x}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}$ M1: Replaces $\text{sech}^2 y$ with $\left(\frac{2}{x}\right)^2$ A1: Correct equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form in terms of x only.		M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1
(a) **Way 5**	$y = \text{arsech}\left(\frac{x}{2}\right) \Rightarrow \text{sech}\, y = \frac{x}{2} \Rightarrow y = \text{artanh}\left(\sqrt{1-\left(\frac{x}{2}\right)^2}\right)$ Changes to "artanh" correctly. **Score this as the second M mark on EPEN.**		M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{1}{2}\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}}{1-\left(1-\frac{x^2}{4}\right)} \times -\frac{x}{2}$ M1: Differentiates to the form $\frac{kx\left(1-\frac{x^2}{4}\right)^{-\frac{1}{2}}}{1-\left(1-\frac{x^2}{4}\right)}$ oe A1: Correct equation involving $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form in terms of x only. **Score this as the first M mark and first A mark on EPEN.**		M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{x\sqrt{4-x^2}}$	Correct derivative in the required form or correct values for p and q.	A1

There may be other methods used.
If you are in any doubt if the method deserves any marks use Review.

(b)	$\mathrm{f}(x)=\tanh^{-1}(x)+\operatorname{sech}^{-1}\left(\frac{x}{2}\right)\Rightarrow \mathrm{f}'(x)=\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}$ Correct f′(x) following through their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ Also allow with "made up" p and q or the letters p and q.		B1ft
	$\frac{1}{1-x^2}-\frac{2}{x\sqrt{4-x^2}}=0\Rightarrow 2\left(1-x^2\right)=x\sqrt{4-x^2}\Rightarrow 4\left(1-x^2\right)^2=x^2\left(4-x^2\right)$ Sets $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ with their (a) of the form $\frac{p}{x\sqrt{q-x^2}}$ and squares both sides to reach a quartic equation		M1
	$5x^4-12x^2+4=0$	Correct quartic	A1
	$5x^4-12x^2+4=0\Rightarrow x^2=2,\ 0.4$ $\Rightarrow x=\ldots$	Solves their quartic equation to obtain a value for x^2 and proceeds to a value for x. Apply usual rules for solving and check if necessary. Allow complex roots.	M1
	$x=\sqrt{\frac{2}{5}}$	Correct exact answer (allow equivalents e.g. $\frac{\sqrt{10}}{5}$). If any extra answers given score A0 e.g. $x=\pm\sqrt{\frac{2}{5}}$	A1
			(5)
			Total 9

Special case:

It is possible for a correct solution in (b) following a sign error in (a) e.g.

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{x\sqrt{4-x^2}}$$

$$\mathrm{f}(x)=\tanh^{-1}(x)+\operatorname{sech}^{-1}\left(\frac{x}{2}\right)\Rightarrow \mathrm{f}'(x)=\frac{1}{1-x^2}+\frac{2}{x\sqrt{4-x^2}}$$

$$\frac{1}{1-x^2}+\frac{2}{x\sqrt{4-x^2}}=0\Rightarrow 2\left(1-x^2\right)=-x\sqrt{4-x^2}\Rightarrow 4\left(1-x^2\right)^2=x^2\left(4-x^2\right)\text{ etc.}$$

This is likely to score M1M1A0A0 in (a) but allow full recovery in (b) if it leads to the correct answer.