Question Number	Scheme	Notes	Marks
4(a)	$\lambda = 3 \Rightarrow \lvert\mathbf{M}-3\mathbf{I}\rvert = \begin{vmatrix} 3 & k & 2 \\ k & 2 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 0 \Rightarrow 3(8)-k(4k)+2(-4)=0$ or e.g. $\lvert\mathbf{M}-\lambda\mathbf{I}\rvert = \begin{vmatrix} 6-\lambda & k & 2 \\ k & 5-\lambda & 0 \\ 2 & 0 & 7-\lambda \end{vmatrix} = 0$ $\Rightarrow (6-\lambda)(5-\lambda)(7-\lambda)-k(k(7-\lambda))+2(0-2(5-\lambda))=0 \Rightarrow 24-k(4k)-8=0$ Correct interpretation of 3 being an eigenvalue leading to the formation of **a quadratic equation in *k* only**. If the method for forming the determinant is not clear then look for at least 2 correct "components". NB rule of Sarrus gives $24 - 8 - 4k^2 = 0$		M1
	$\Rightarrow 4k^2 = 16 \Rightarrow k = ...$	Solves quadratic. **Depends on the first M.**	**d**M1
	$k = \pm 2$	Correct values	A1
			(3)
(a) **Way 2**	$\begin{pmatrix} 6 & k & 2 \\ k & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} 6x+ky+2z=3x \\ kx+5y=3y \\ 2x+7z=3z \end{matrix}$ $z = -\frac{1}{2}x,\ y = -\frac{1}{2}kx \Rightarrow 6x - \frac{k^2x}{2} - x = 3x \Rightarrow \frac{k^2}{2} = 2$ Eliminates *z* and *y* and reaches a quadratic equation in *k* only		M1
	$\frac{k^2}{2} = 2 \Rightarrow k = ...$	Solves quadratic. **Depends on the first M.**	**d**M1
	$k = \pm 2$	Correct values	A1
(b)	$k = -2 \Rightarrow \lvert\mathbf{M}-\lambda\mathbf{I}\rvert = \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 5-\lambda & 0 \\ 2 & 0 & 7-\lambda \end{vmatrix}$ $\Rightarrow (6-\lambda)(7-\lambda)(5-\lambda)+2(2\lambda-14)+2(2\lambda-10)=0$ Applies a value of *k* from (a) and a recognisable attempt at the characteristic equation (the "= 0" is not needed here). If the method is not clear then look for at least 2 correct "components".		M1
	$\Rightarrow \lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0 \Rightarrow \lambda = ...$	Solves cubic. May use $\lambda = 3$ as a factor or calculator to solve. **Depends on the first mark.** Allow complex roots.	**d**M1
	$\lambda = 6,\ 9\ (,3)$	Correct values. Allow to come from $k = 2$	A1
			(3)

(c)	$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 5 & 0 \\ 2 & 0 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{matrix} 6x-2y+2z=3x \\ -2x+5y=3y \\ 2x+7z=3z \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ or $\begin{pmatrix} 3 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} 6x-2y+2z=0 \\ -2x+5y=0 \\ 2x+7z=0 \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$ Correct strategy for finding the eigenvector using a value of k from (a) Note that the cross product of any 2 rows or columns of $\mathbf{M} - 3\mathbf{I}$ gives an eigenvector		M1
	$p\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct eigenvector	A1
	$\frac{1}{3}\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	Any correct normalised eigenvector	A1
			(3)
			Total 9