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6. $$I_n = \int e^x \sin^n x \, dx \qquad n \in \mathbb{Z} \qquad n \geqslant 0$$

(a) Show that

$$I_n = \frac{e^x \sin^{n-1} x}{n^2 + 1}(\sin x - n\cos x) + \frac{n(n-1)}{n^2 + 1} I_{n-2} \qquad n \geqslant 2$$

(6)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{2}} e^x \sin^4 x \, dx$$

giving your answer in the form $Ae^{\frac{\pi}{2}} + B$ where A and B are rational numbers to be determined.

(4)

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(Total 10 marks)

Q6