Question Number	Scheme	Notes	Marks
8(a)	$b^2 = a^2\left(1-e^2\right) \Rightarrow 4 = 9\left(1-e^2\right) \Rightarrow e = ...$ or e.g. $e = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow e = ...$	Uses a correct formula with a and b correctly placed to find a value for e	M1
	$e = \frac{\sqrt{5}}{3}$	Correct value (or equivalent) $e = \pm\frac{\sqrt{5}}{3}$ scores A0	A1
			(2)
(b)(i)	$\left(\pm ae,\ 0\right) = \left(\pm\sqrt{5},\ 0\right)$ or $\left(\pm 3\frac{\sqrt{5}}{3},\ 0\right)$ Correct foci. Must be coordinates but allow unsimplified and isw if necessary. Follow through their e so allow for $(\pm 3\times$their e, 0)		B1ft
(ii)	$x = \pm\frac{a}{e} = \pm\frac{9}{\sqrt{5}}$ or $x = \pm\frac{3}{\frac{\sqrt{5}}{3}}$ Correct directrices. Must be equations but allow unsimplified and isw if necessary. Follow through their e so allow for $x = \pm 3$/their e		B1ft
			(2)
	Special case: Use of a^2 for a and b^2 for b **consistently** scores M0A0 in (a) and B1ft B1ft in (b) This gives $e = \frac{\sqrt{65}}{9}$, $\left(\pm\sqrt{65},\ 0\right)$, $x = \pm\frac{81}{\sqrt{65}}$		
(c)	$\frac{dx}{d\theta} = -3\sin\theta,\ \frac{dy}{d\theta} = 2\cos\theta$ or $\frac{2x}{9} + \frac{2y}{4}\frac{dy}{dx} = 0$ or $y = \left(4-\frac{4x^2}{9}\right)^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = -\frac{4x}{9}\left(4-\frac{4x^2}{9}\right)^{-\frac{1}{2}}$ $\Rightarrow \frac{dy}{dx} = ...\left(= \frac{2\cos\theta}{-3\sin\theta}\right)$	Correct strategy for the gradient of l in terms of θ. Allow $\frac{dy}{dx} = \frac{2\cos\theta}{-3\sin\theta}$ to be stated.	M1
	$y - 2\sin\theta = \frac{2\cos\theta}{-3\sin\theta}\left(x - 3\cos\theta\right)$	Correct straight line method (any complete method). Finding the equation of the normal is M0.	M1
	$-3y\sin\theta + 6\sin^2\theta = 2x\cos\theta - 6\cos^2\theta$ $2x\cos\theta + 3y\sin\theta = 6$*	Cso with at least one intermediate line of working	A1*
			(3)

(d)	$l_2: \quad y = \dfrac{3\sin\theta}{2\cos\theta}x$	Correct equation for l_2	B1
	$2x\cos\theta + 3y\sin\theta = 6, y = \dfrac{3\sin\theta}{2\cos\theta}x$ $\Rightarrow x = ..., y = ...$	Complete method for Q	M1
	$Q: \left(\dfrac{12\cos\theta}{4\cos^2\theta + 9\sin^2\theta}, \dfrac{18\sin\theta}{4\cos^2\theta + 9\sin^2\theta} \right)$ Correct coordinates. Allow as $x = ..., y = ...$ and allow equivalent correct expressions as long as they are single fractions e.g. $x = \dfrac{12\cos\theta}{4 + 5\sin^2\theta} \quad y = \dfrac{18\sin\theta}{4 + 5\sin^2\theta}, \quad x = \dfrac{12\cos\theta}{9 - 5\cos^2\theta} \quad y = \dfrac{18\sin\theta}{9 - 5\cos^2\theta}$		A1
			(3)

(e)	At Q, $\dfrac{y}{x}=\dfrac{3}{2}\tan\theta$	Uses their coordinates of Q to attempt an equation connecting x, y and θ or states or uses the equation found in (d)	M1
	$x=\dfrac{12\cos\theta}{4\cos^2\theta+9\sin^2\theta}=\dfrac{12\sec\theta}{4+9\tan^2\theta}\Rightarrow x^2=\dfrac{144\sec^2\theta}{\left(4+9\tan^2\theta\right)^2}=\dfrac{144\left(1+\dfrac{4y^2}{9x^2}\right)}{\left(4+9\times\dfrac{4y^2}{9x^2}\right)^2}$ or $y=\dfrac{18\sin\theta}{4\cos^2\theta+9\sin^2\theta}=\dfrac{12\sec\theta\tan\theta}{4+9\tan^2\theta}$ $\Rightarrow y^2=\dfrac{324\sec^2\theta\tan^2\theta}{\left(4+9\tan^2\theta\right)^2}=\dfrac{324\left(1+\dfrac{4y^2}{9x^2}\right)\dfrac{4y^2}{9x^2}}{\left(4+9\times\dfrac{4y^2}{9x^2}\right)^2}$ Eliminates θ **Depends on the first mark.**		**d**M1
	$\Rightarrow x^2=\dfrac{x^2\left(9x^2+4y^2\right)}{\left(x^2+y^2\right)^2}\Rightarrow\left(x^2+y^2\right)^2=9x^2+4y^2$ or $\Rightarrow 9\times16x^2y^2\left(1+\dfrac{y^2}{x^2}\right)^2=4\times18^2\left(1+\dfrac{4y^2}{9x^2}\right)\Rightarrow\left(x^2+y^2\right)^2=9x^2+4y^2$ Correct equation or correct values for α and β.		A1
			(3)
(e) **Way 2**	$x=\dfrac{12\cos\theta}{4+5\sin^2\theta}\quad y=\dfrac{18\sin\theta}{4+5\sin^2\theta}\Rightarrow\left(x^2+y^2\right)^2=\left(\dfrac{144\cos^2\theta+324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2$ Uses their Q to obtain an expression for $\left(x^2+y^2\right)^2$ in terms of θ		M1
	$\left(\dfrac{144\cos^2\theta+324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2=\left(\dfrac{144+180\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\right)^2=\left(\dfrac{36\left(4+5\sin^2\theta\right)}{\left(4+5\sin^2\theta\right)^2}\right)^2=\dfrac{1296}{\left(4+5\sin^2\theta\right)^2}$ $\dfrac{1296}{\left(4+5\sin^2\theta\right)^2}=\alpha x^2+\beta y^2=\alpha\dfrac{144\cos^2\theta}{\left(4+5\sin^2\theta\right)^2}+\beta\dfrac{324\sin^2\theta}{\left(4+5\sin^2\theta\right)^2}\Rightarrow\alpha=\ldots,\beta=\ldots$ Substitutes into the given answer and solves for α and β **Depends on the first mark.**		**d**M1
	$\left(x^2+y^2\right)^2=9x^2+4y^2$	Correct expression or correct values for α and β.	A1
			Total 13

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