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8. The ellipse E has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Determine the eccentricity of E

(2)

(b) Hence, for this ellipse, determine

(i) the coordinates of the foci,

(ii) the equations of the directrices.

(2)

The point P lies on E and has coordinates $(3\cos\theta, 2\sin\theta)$.

The line l_1 is the tangent to E at the point P

(c) Using calculus, show that an equation for l_1 is

$$2x\cos\theta + 3y\sin\theta = 6$$

(3)

The line l_2 passes through the origin and is perpendicular to l_1

The line l_1 intersects the line l_2 at the point Q

(d) Determine the coordinates of Q

(3)

(e) Show that, as θ varies, the point Q lies on the curve with equation

$$(x^2 + y^2)^2 = \alpha x^2 + \beta y^2$$

where α and β are constants to be determined.

(3)

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