Question Number	Scheme	Notes	Marks
1(a)	$(\cosh A\cosh B+\sinh A\sinh B=)\left(\frac{e^{A}+e^{-A}}{2}\right)\left(\frac{e^{B}+e^{-B}}{2}\right)+\left(\frac{e^{A}-e^{-A}}{2}\right)\left(\frac{e^{B}-e^{-B}}{2}\right)$ $=\frac{e^{A+B}+e^{A-B}+e^{B-A}+e^{-A-B}+e^{A+B}-e^{A-B}-e^{B-A}+e^{-A-B}}{4}$ Expresses the lhs in terms of exponentials correctly, combines terms and combines fractions with common denominator (Brackets not needed due to fraction lines)		M1
	$=\frac{2e^{A+B}+2e^{-(A+B)}}{4}=\frac{e^{A+B}+e^{-(A+B)}}{2}=\cosh(A+B)*$ Fully correct proof with no errors		A1*
			(2)
(b)	$\cosh(x+\ln 2)=\cosh x\cosh(\ln 2)+\sinh x\sinh(\ln 2)$ $=\left(\frac{2+\frac{1}{2}}{2}\right)\cosh x+\left(\frac{2-\frac{1}{2}}{2}\right)\sinh x$ Applies the result from part (a) and evaluates both cosh(ln2) and sinh(ln2) **Use of (a) must be seen**		M1
	$\frac{5}{4}\cosh x+\frac{3}{4}\sinh x=5\sinh x$ $\Rightarrow\frac{5}{4}\cosh x=\frac{17}{4}\sinh x$	Collects terms and reaches $a\cosh x=b\sinh x$ oe Depends on the first M mark	dM1
	$5\cosh x=17\sinh x$ oe	Correct equation	A1
	$x=\frac{1}{2}\ln\left(\frac{1+\frac{5}{17}}{1-\frac{5}{17}}\right)$ Or $\frac{e^{2x}-1}{e^{2x}+1}=\frac{5}{17}\Rightarrow x=...$	Moves to tanh x and uses the correct logarithmic form for artanh$x$ or reverts to exponential forms and solves for x Depends on both M marks	ddM1
	$x=\frac{1}{2}\ln\left(\frac{11}{6}\right)$	Cao $\left(\text{Accept integer multiples of } \frac{11}{6}\right)$	A1
			(5)
			Total 7

Way 2			
(b)	$\cosh(x+\ln 2)=\cosh x\cosh(\ln 2)+\sinh x\sinh(\ln 2)$ $=\left(\frac{2+\frac{1}{2}}{2}\right)\cosh x+\left(\frac{2-\frac{1}{2}}{2}\right)\sinh x$ Applies the result from part (a) and evaluates both cosh(ln2) and sinh(ln2) **Use of (a) must be seen**		M1
	$\Rightarrow 5\cosh x=17\sinh x$ dM1: Collects terms and reaches an equation of form $A\cosh x=B\sinh x$ A1: Correct equation		dM1A1
	$5\left(\frac{e^{x}+e^{-x}}{2}\right)=17\left(\frac{e^{x}-e^{-x}}{2}\right)$		
	$12e^{x}=22e^{-x}\Rightarrow e^{2x}=\frac{22}{6}\Rightarrow x=...$	Changes to exponentials (correct forms) And solves for x	ddM1
	$x=\frac{1}{2}\ln\left(\frac{11}{6}\right)$	Cao $\left(\text{Accept integer multiples of } \frac{11}{6}\right)$	A1
Way 3			
	$\cosh(x+\ln 2)=\cosh x\cosh(\ln 2)+\sinh x\sinh(\ln 2)$ $\left(\frac{e^{x}+e^{-x}}{2}\right)\left(\frac{e^{\ln 2}+e^{-\ln 2}}{2}\right)+\left(\frac{e^{x}-e^{-x}}{2}\right)\left(\frac{e^{\ln 2}-e^{-\ln 2}}{2}\right)=5\left(\frac{e^{x}-e^{-x}}{2}\right)$ Applies the result from part (a) and uses the exponential forms of the hyperbolic functions. **Use of (a) must be seen**		M1
	eg $5e^{x}+5e^{-x}=17e^{x}-17e^{-x}$ oe	Evaluates $e^{\ln 2}$ and $e^{-\ln 2}$ and starts to collect terms	dM1
	$12e^{2x}=22\Rightarrow e^{2x}=\frac{11}{6}$	Correct value for e^{2x}	A1
	x = ...	Solves for x	ddM1
	$x=\frac{1}{2}\ln\left(\frac{11}{6}\right)$	Cao $\left(\text{Accept integer multiples of } \frac{11}{6}\right)$	A1

NB: Squaring and obtaining a value for sinh*x* or cosh*x* introduces extra answers. If these extra answers are then eliminated M1A1 is available but if no attempt at elimination is made award M0A0