**3.** $$\mathbf{M} = \begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix}$$

Given that $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is an eigenvector of $\mathbf{M}$,

(a) determine the corresponding eigenvalue.

**(1)**

Given that 8 is an eigenvalue of $\mathbf{M}$,

(b) determine a corresponding eigenvector.

**(2)**

(c) Determine a diagonal matrix $\mathbf{D}$ and an orthogonal matrix $\mathbf{P}$ such that

$$\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P}$$

**(5)**

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**Question 3 continued**

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**Question 3 continued**

**(Total for Question 3 is 8 marks)**