Question Number	Scheme	Notes	Marks
7.	$I_n = \int \frac{x^n}{\sqrt{10-x^2}}\,\mathrm{d}x \qquad n \in \mathbb{N},\ \lvert x\rvert < \sqrt{10}$		
(a)	$I_n = \int \frac{x^n}{\sqrt{10-x^2}}\,\mathrm{d}x = \int \frac{x^{n-1}\times x}{\sqrt{10-x^2}}\,\mathrm{d}x$	Writes x^n as $x\times x^{n-1}$	M1
	$\int \frac{x^{n-1}\times x}{\sqrt{10-x^2}}\,\mathrm{d}x = -x^{n-1}\left(10-x^2\right)^{\frac{1}{2}} + (n-1)\int x^{n-2}\left(10-x^2\right)^{\frac{1}{2}}\,\mathrm{d}x$ **d**M1: Uses integration by parts to obtain $\int \frac{x^{n-1}\times x}{\sqrt{10-x^2}}\,\mathrm{d}x = \alpha x^{n-1}\left(10-x^2\right)^{\frac{1}{2}} + \beta\int x^{n-2}\left(10-x^2\right)^{\frac{1}{2}}\,\mathrm{d}x$ A1: Correct expression		**d**M1A1
	$= \ldots + (n-1)\int x^{n-2}\left(10-x^2\right)\left(10-x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x$ $= \ldots + 10(n-1)\int x^{n-2}\left(10-x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x - (n-1)\int x^{n}\left(10-x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x$ Applies $\left(10-x^2\right)^{\frac{1}{2}} = \left(10-x^2\right)\left(10-x^2\right)^{-\frac{1}{2}}$ and splits into 2 integrals		**d**M1
	$= \ldots + 10(n-1)I_{n-2} - (n-1)I_n \Rightarrow nI_n$	Introduces I_{n-2} and I_n and makes progress to the given result	**d**M1
	$nI_n = 10(n-1)I_{n-2} - x^{n-1}\left(10-x^2\right)^{\frac{1}{2}}$ * Fully correct proof with no errors (recovery of missing brackets counts as an error) as does missing dx		A1*
			(6)
(b)	$I_1 = \int_0^1 \frac{x}{\sqrt{10-x^2}}\,\mathrm{d}x = \left[-\left(10-x^2\right)^{\frac{1}{2}}\right]_0^1 \left(= -3+\sqrt{10}\right)$ Correct method for I_1 Limits can be substituted later		M1
	$5I_5 = 10\times 4I_3 + \ldots$	Applies the reduction formula at least once Allow with 3 or $\left[-x^4\left(10-x^2\right)^{\frac{1}{2}}\right]_0^1$	M1
	$I_5 = 8I_3 - \frac{3}{5} = 8\left(\frac{20}{3}I_1 - 1\right) - \frac{3}{5} = \frac{160}{3}I_1 - \frac{43}{5}$ $I_5 = \frac{160}{3}\left(\sqrt{10}-3\right) - \frac{43}{5}$ Completes the process using their I_1 to obtain a numerical value for I_5 Limits must now be substituted		M1
	$= \frac{1}{15}\left(800\sqrt{10} - 2529\right)$	Cao	A1
			(4)
			Total 10