7. $$I_n = \int \frac{x^n}{\sqrt{10 - x^2}}\,\mathrm{d}x \qquad n \in \mathbb{N} \qquad |x| < \sqrt{10}$$

(a) Show that

$$nI_n = 10(n-1)I_{n-2} - x^{n-1}(10 - x^2)^{\frac{1}{2}} \qquad n \geqslant 2$$

(6)

(b) Hence find the exact value of

$$\int_0^1 \frac{x^5}{\sqrt{10 - x^2}}\,\mathrm{d}x$$

giving your answer in the form $\frac{1}{15}\left(p\sqrt{10} + q\right)$ where p and q are integers to be determined.

(4)

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(Total for Question 7 is 10 marks)

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