Question Number	Scheme	Notes	Marks
3 **Way 1** **Converts to sinh and cosh**	$4\tanh x - \operatorname{sech} x = 1$ $4\dfrac{\sinh x}{\cosh x} - \dfrac{1}{\cosh x} = 1$ $4\sinh x - 1 - \cosh x = 0$ $4\dfrac{e^x - e^{-x}}{2} - 1 - \dfrac{e^x + e^{-x}}{2} = 0$	Replaces **one** hyperbolic function with its correct exponential equivalent. Allow for correct replacement of just e.g., $\sinh x$ after using $\tanh x = \dfrac{\sinh x}{\cosh x}$ May follow errors but do not allow any further marks if the original equation was reduced to one in a single hyperbolic function.	M1
	$3e^{2x} - 2e^x - 5 = 0$	M1: Obtains an equation which if terms are collected is a 3TQ (or 2TQ with no constant) in e^x A1: Correct 3TQ	M1 A1
	$e^x = \dfrac{2 \pm \sqrt{4+60}}{6}$ $\left(\Rightarrow \dfrac{2+8}{6} = \dfrac{5}{3}\right)$	M1: Solves 3TQ (or 2TQ with no constant) in e^x. Apply usual rules. If no working seen they must achieve one correct root of their equation to 3sf which may be complex. If 2TQ must get a correct non-zero root of their equation. A1: Any correct unsimplified expression for e^x that includes the positive root. Must be exact	M1 A1
	$x = \ln\dfrac{5}{3}$	$\ln\dfrac{5}{3}$, $\ln 1\dfrac{2}{3}$, $\ln 1.\dot{6}$ only but allow k = ... No unrejected extra solutions	A1
			Total 6
Way 2 **Straight to e^x**	$4\dfrac{e^x - e^{-x}}{e^x + e^{-x}} - \dfrac{2}{e^x + e^{-x}} = 1$	Replaces **one** hyperbolic function with its correct exponential equivalent	M1
	$3e^{2x} - 2e^x - 5 = 0$	M1: Obtains an equation which if terms are collected is a 3TQ (or 2TQ with no constant) in e^x A1: Correct 3TQ	M1 A1
	$e^x = \dfrac{2 \pm \sqrt{4+60}}{6}$ $\left(\Rightarrow \dfrac{2+8}{6} = \dfrac{5}{3}\right)$	M1: Solves 3TQ (or 2TQ with no constant) in e^x. Apply usual rules. If no working seen they must achieve one correct root of their equation to 3sf which may be complex. If 2TQ must get a correct non-zero root of their equation. A1: Any correct unsimplified expression for e^x that includes the positive root. Must be exact	M1 A1
	$x = \ln\dfrac{5}{3}$	$\ln\dfrac{5}{3}$, $\ln 1\dfrac{2}{3}$, $\ln 1.\dot{6}$ only but allow k = ... No unrejected extra solutions	A1
			Total 6
	In Ways 1 & 2, if they form an equation which is not a quadratic in e^x they must achieve the correct exact root of $\dfrac{5}{3}$ to access the middle four marks		

Question Number	Scheme	Notes	Marks
3 **Way 3a** **Squaring (sinh)**	$4\sinh x - 1 = \cosh x$ $16\sinh^2 x - 8\sinh x + 1 = \cosh^2 x$ $16\sinh^2 x - 8\sinh x + 1 = 1 + \sinh^2 x$	Squares (condone poor squaring) and uses a correct hyperbolic identity to obtain a quadratic equation in sinh *x*	M1
	$15\sinh^2 x - 8\sinh x = 0$	M1: Obtains a 2TQ with no constant or 3TQ in sinh *x* A1: Correct 2TQ	M1 A1
	$\sinh x = \frac{8}{15}$	Solves 2TQ (with no constant) or 3TQ in sinh *x*. Apply usual rules. If 2TQ must get a correct non-zero root of their equation. If no working seen they must achieve one correct root of their equation to 3sf which may be complex.	M1
	$x = \text{arsinh}\frac{8}{15} = \ln\left(\frac{8}{15} + \sqrt{\left(\frac{8}{15}\right)^2 + 1}\right)$ or $15e^{2x} - 16e^x - 15 = 0 \Rightarrow$ $e^x = \frac{16 \pm \sqrt{256 + 900}}{30}$	A correct unsimplified expression for *x* as a ln (or any correct unsimplified expression for e^x if they revert to exponentials). Must be exact	A1
	$x = \ln\frac{5}{3}$	$\ln\frac{5}{3}$, $\ln 1\frac{2}{3}$, $\ln 1.\dot{6}$ only but allow *k* = ... No unrejected extra solutions	A1
			Total 6
Way 3b **Squaring (sech)**	$4\tanh x = 1 + \text{sech}\, x$ $16\tanh^2 x = 1 + 2\,\text{sech}\, x + \text{sech}^2 x$ $16\left(1 - \text{sech}^2 x\right) = 1 + 2\,\text{sech}\, x + \text{sech}^2 x$	Squares (condone poor squaring) and uses a correct hyperbolic identity to obtain a quadratic equation in sech *x*	M1
	$17\,\text{sech}^2 x + 2\,\text{sech}\, x - 15 = 0$	M1: Obtains a 2TQ (with no constant) or 3TQ in sech *x* A1: Correct 3TQ	M1 A1
	$\left(17\,\text{sech}\, x - 15\right)(\text{sech}\, x + 1) = 0$ $\text{sech}\, x = \frac{15}{17}$	Solves 2TQ with no constant or 3TQ in sech *x*. Apply usual rules. If 2TQ must get a correct non-zero root of their equation. If no working seen they must achieve one correct root of their equation to 3sf which may be complex.	M1
	$x = \text{arcosh}\frac{17}{15} = \ln\left(\frac{17}{15} + \sqrt{\left(\frac{17}{15}\right)^2 - 1}\right)$ or $15e^{2x} - 34e^x + 15 = 0 \Rightarrow$ $e^x = \frac{34 \pm \sqrt{1156 - 900}}{30}$	A correct unsimplified expression for *x* as a ln (or any correct unsimplified expression for e^x if they revert to exponentials). Must be exact	A1
	$x = \ln\frac{5}{3}$	$\ln\frac{5}{3}$, $\ln 1\frac{2}{3}$, $\ln 1.\dot{6}$ only but allow *k* = ... No unrejected extra solutions	A1
			Total 6

Question Number	Scheme	Notes	Marks
3 **Way 3c** **Squaring (tanh)**	$4\tanh x - 1 = \operatorname{sech} x$ $16\tanh^2 x - 8\tanh x + 1 = \operatorname{sech}^2 x$ $16\tanh^2 x - 8\tanh x + 1 = 1 - \tanh^2 x$	Squares (condone poor squaring) and uses a correct hyperbolic identity to obtain a quadratic equation in tanh *x*	M1
	$17\tanh^2 x - 8\tanh x = 0$	M1: Obtains a 2TQ with no constant or 3TQ in tanh *x* A1: Correct 2TQ	M1 A1
	$\tanh x = \dfrac{8}{17}$	Solves 2TQ with no constant or 3TQ in tanh *x*. Apply usual rules. If 2TQ must get a correct non-zero root of their equation. If no working seen they must achieve one correct root of their equation to 3sf which may be complex.	M1
	$x = \operatorname{artanh}\dfrac{8}{17} = \dfrac{1}{2}\ln\left(\dfrac{1+\frac{8}{17}}{1-\frac{8}{17}}\right)$ or $9e^{2x} - 25 = 0 \Rightarrow$ $e^x = \dfrac{5}{3}$	A correct unsimplified expression for *x* as a ln (or any correct unsimplified expression for e^x if they revert to exponentials). Must be exact	A1
	$x = \ln\dfrac{5}{3}$	$\ln\dfrac{5}{3}$, $\ln 1\dfrac{2}{3}$, $\ln 1.\dot{6}$ only but allow *k* = ... No unrejected extra solutions	A1
			Total 6
Way 3d **Squaring (cosh)**	$4\sinh x = 1 + \cosh x$ $16\sinh^2 x = 1 + 2\cosh x + \cosh^2 x$ $16\cosh^2 x - 16 = 1 + 2\cosh x + \cosh^2 x$	Squares (condone poor squaring) and uses a correct hyperbolic identity to obtain a quadratic equation in cosh *x*	M1
	$15\cosh^2 x - 2\cosh x - 17 = 0$	M1: Obtains a 2TQ with no constant or 3TQ in cosh *x* A1: Correct 3TQ	M1 A1
	$(15\cosh x - 17)(\cosh x + 1) = 0$ $\cosh x = \dfrac{17}{15}$	Solves 2TQ (with no constant) or 3TQ in cosh *x*. Apply usual rules. If 2TQ must get a correct non-zero root of their equation. If no working seen they must achieve one correct root of their equation to 3sf which may be complex.	M1
	$x = \operatorname{arcosh}\dfrac{17}{15} = \ln\left(\dfrac{17}{15} + \sqrt{\left(\dfrac{17}{15}\right)^2 - 1}\right)$ or $15e^{2x} - 34e^x + 15 = 0 \Rightarrow$ $e^x = \dfrac{34 \pm \sqrt{1156 - 900}}{30}$	A correct unsimplified expression for *x* as a ln (or any correct unsimplified expression for e^x if they revert to exponentials). Must be exact	A1
	$x = \ln\dfrac{5}{3}$	$\ln\dfrac{5}{3}$, $\ln 1\dfrac{2}{3}$, $\ln 1.\dot{6}$ only but allow *k* = ... No unrejected extra solutions	A1
			Total 6