Question Number	Scheme	Notes	Marks
4(a)	$\int \frac{1}{\sqrt{9x^2+16}}\,\mathrm{d}x = \frac{1}{3}\int \frac{1}{\sqrt{x^2+\frac{16}{9}}}\,\mathrm{d}x$ $= \frac{1}{3}\text{arsinh}\left(\frac{3x}{4}\right)$ or $\frac{1}{3}\text{arsinh}\left(\frac{x}{\frac{4}{3}}\right)$ $(+c)$ or $\frac{1}{3}\ln\left(x+\sqrt{x^2+\left(\frac{4}{3}\right)^2}\right)$ $(+c)$	M1: Obtains $p\,\text{arsinh}(qx)$ or $r\ln\left\{x+\sqrt{x^2+s}\right\}$ or $t\ln\left(ux+\sqrt{vx^2+w}\right)$ $p, q, r, s, t, u, v, w > 0$ A1: Any correct expression. Could be unsimplified and isw. The "+c" is not required. Allow $\sinh^{-1}$ and condone "arcsinh". "arcsin" or "arsin" is M0	M1 A1
			(2)
(b)	$\int_{-2}^{2} \frac{1}{\sqrt{9x^2+16}}\,\mathrm{d}x$ $= \left[\frac{1}{3}\text{arsinh}\left(\frac{3x}{4}\right)\right]_{-2}^{2}$ or $\left[\frac{2}{3}\text{arsinh}\left(\frac{3x}{4}\right)\right]_{0}^{2}$ $= \frac{1}{3}\text{arsinh}\left(\frac{3\times 2}{4}\right) - \frac{1}{3}\text{arsinh}\left(\frac{3\times -2}{4}\right)$ or $\frac{2}{3}\text{arsinh}\left(\frac{3}{2}\right)$ OR $\left[\frac{1}{3}\ln\left(x+\sqrt{x^2+\frac{16}{9}}\right)\right]_{-2}^{2}$ $= \frac{1}{3}\ln\left(2+\sqrt{2^2+\frac{16}{9}}\right) - \frac{1}{3}\ln\left(-2+\sqrt{(-2)^2+\frac{16}{9}}\right)$ or $\frac{2}{3}\left(\ln\left(2+\sqrt{2^2+\frac{16}{9}}\right) - \ln\left(0+\sqrt{0^2+\frac{16}{9}}\right)\right)$	Substitutes the limits 2 and -2 into an expression of the form $p\,\text{arsinh}(qx)$ or $r\ln\left\{x+\sqrt{x^2+s}\right\}$ or $t\ln\left(ux+\sqrt{vx^2+w}\right)$ $p, q, r, s, t, u, v, w > 0$ and subtracts either way round or obtains an expression for $2[...]_0^{\pm 2}$ The expression does not have to be consistent with their answer to (a). No rounded decimals unless exact values recovered. Any f(0) = 0 can be implied by omission. Condone poor bracketing.	M1
	$\frac{1}{3}\ln\left(\frac{11}{2}+\frac{3\sqrt{13}}{2}\right)$ or $\frac{1}{3}\ln\frac{11+3\sqrt{13}}{2}$ or $\frac{2}{3}\ln\left(\frac{3}{2}+\frac{\sqrt{13}}{2}\right)$ or $\frac{2}{3}\ln\frac{3+\sqrt{13}}{2}$	**d**M1: Obtains an expression of the form $a\ln\left(b+c\sqrt{13}\right)$ *or* $a\ln\left(\frac{d+e\sqrt{13}}{f}\right)$ where a, b, c, d, e, f are exact and > 0. Condone poor bracketing. **Requires previous M mark.** A1: Any correct equivalent in an appropriate form (fractions may not be in simplest form) with correct bracketing if necessary and isw. **Must come from correct work**. Allow e.g., $a=\frac{2}{3}$, $b=\frac{3}{2}$, $c=\frac{1}{2}$	**d**M1 A1
	For information the decimal answer is 0.7965038115		**(3)**
			Total 5