Question Number	Scheme	Notes	Marks
5(a)	$\begin{vmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{vmatrix}$ $= a(4\times5-0)-a(-5a-0)+1\left(-a^2-(4\times4)\right)$	Uses a correct method for det **A** (implied by two correct parts) to obtain an expression in a	M1
	$\Rightarrow 20a+5a^2-a^2-16=0$ $\Rightarrow a^2+5a-4=0$ $\Rightarrow a=\dfrac{-5+\sqrt{41}}{2}$	Correct exact value oe Condone $\dfrac{-5\pm\sqrt{41}}{2}$	A1
			(2)
(b)(i) **Way 1** $\lvert\mathbf{A}-\lambda\mathbf{I}\rvert$	$\lvert\mathbf{A}-\lambda\mathbf{I}\rvert = \begin{vmatrix} a-\lambda & a & 1 \\ -a & 4-\lambda & 0 \\ 4 & a & 5-\lambda \end{vmatrix}$ $=(a-\lambda)(4-\lambda)(5-\lambda)-a\times-a(5-\lambda)+\left(-a^2-4(4-\lambda)\right)$ or $\lvert\mathbf{A}-2\mathbf{I}\rvert = \begin{vmatrix} a-2 & a & 1 \\ -a & 2 & 0 \\ 4 & a & 3 \end{vmatrix}$ $=6(a-2)-a\times-3a+\left(-a^2-8\right)$	Obtains an expression for $\lvert\mathbf{A}-\lambda\mathbf{I}\rvert$ in terms of a and λ or just a if λ is replaced by 2. If method unclear insist on 2 out of 3 correct parts. May multiply along any row/column. Sarrus leads to the same expressions shown (or the expressions all multiplied by -1 if "=0").	M1
	$\lambda=2\Rightarrow(a-2)\times2\times3+3a^2-a^2-8=0$ $2a^2+6a-20=0 \Rightarrow a^2+3a-10=0$ $\Rightarrow (a-2)(a+5)=0 \Rightarrow a=...$	Following use of $\lambda=2$, forms and solves a 3TQ in a. Apply usual rules. If no working they must obtain one correct solution for their 3TQ which could be complex. Could be implied. **Requires previous M mark.**	**d**M1
	$(a>0 \Rightarrow) a=2$	Correct value of a from correct work. If –5 is offered imply its rejection if 2 alone is used in (ii)	A1
	If a = 2 is arrived at fortuitously, all marks are available for the remainder of the question		**(3)**
(b)(i) **Way 2** $\mathbf{Ax}=2\mathbf{x}$	$\mathbf{Ax}=2\mathbf{x}\Rightarrow$ $ax+ay+z=2x$ $-ax+4y=2y$ $4x+ay+5z=2z$	Uses $\mathbf{Ax}=2\mathbf{x}$ $\left[or\left(\mathbf{A}-2\mathbf{I}\right)\mathbf{x}=0\right]$ to obtain three simultaneous equations. Allow if given as two equal vectors.	M1
	$\Rightarrow a^2+3a-10=0$ $\Rightarrow (a-2)(a+5)=0 \Rightarrow a=...$	Forms and solves a 3TQ in a. Apply usual rules. If calculator used must obtain one correct solution for their 3TQ which could be complex. Could be implied. **Requires previous M mark.**	**d**M1
	$(a>0 \Rightarrow) a=2$	Correct value of a from correct work. If –5 is offered imply its rejection if 2 alone is used in (ii)	A1
	If a = 2 is arrived at fortuitously, all marks are available for the remainder of the question		**(3)**

Question Number	Scheme	Notes	Marks
5(b)(ii)	$(2-\lambda)(4-\lambda)(5-\lambda)+4(5-\lambda)+(-4-16+4\lambda)=0$ $\Rightarrow (5-\lambda)\left[(2-\lambda)(4-\lambda)+4-4\right]=0$ $\Rightarrow (5-\lambda)(2-\lambda)(4-\lambda)=0 \Rightarrow \lambda = ...$	Uses their value of a in a recognisable attempt at a characteristic equation and achieves a real non-zero eigenvalue $\neq 2$. There must be some algebra but it may be poor.	M1
	4 and 5	Both correct (no extra) and from correct work	A1
	For information the cubic is $\pm\left(\lambda^3-11\lambda^2+38\lambda-40\right)=0$		**(2)**
(c)	$\mathbf{A}\begin{pmatrix}x\\y\\z\end{pmatrix}="4"\begin{pmatrix}x\\y\\z\end{pmatrix}$ *or* $(\mathbf{A}-"4"\mathbf{I})\begin{pmatrix}x\\y\\z\end{pmatrix}=0 \Rightarrow$ $\begin{matrix}2x+2y+z=4x\\-2x+4y=4y\\4x+2y+5z=4z\end{matrix}$ *or* $\begin{matrix}-2x+2y+z=0\\-2x=0\\4x+2y+z=0\end{matrix}$ OR $\mathbf{A}\begin{pmatrix}x\\y\\z\end{pmatrix}="5"\begin{pmatrix}x\\y\\z\end{pmatrix}$ *or* $(\mathbf{A}-"5"\mathbf{I})\begin{pmatrix}x\\y\\z\end{pmatrix}=0 \Rightarrow$ $\begin{matrix}2x+2y+z=5x\\-2x+4y=5y\\4x+2y+5z=5z\end{matrix}$ *or* $\begin{matrix}-3x+2y+z=0\\-2x-y=0\\4x+2y+z=0\end{matrix}$ Uses $\mathbf{Ax}=\lambda\mathbf{x}$ *or* $(\mathbf{A}-\lambda\mathbf{I})\mathbf{x}=0$ with their value of a and a real non-zero value of $\lambda \neq 2$ to obtain three simultaneous equations (allow if given as two equal vectors) Alternatively attempts vector product of two rows of $\mathbf{A}-"4"\mathbf{I}$		M1
	$\pm\begin{pmatrix}0\\-1\\2\end{pmatrix}$ **or** $\pm\begin{pmatrix}1\\-2\\7\end{pmatrix}$	One correct eigenvector. As shown or multiple or with components multiplied by e.g. "k" Accept e.g., $x=0,\ y=-1,\ z=2$	A1
	$\pm\begin{pmatrix}0\\-1\\2\end{pmatrix}$ **and** $\pm\begin{pmatrix}1\\-2\\7\end{pmatrix}$	Both correct eigenvectors. As shown or multiple or with components multiplied by e.g. k Accept $x=...,\ y=...,\ z=...$ Both these 2 A marks could be implied by their normalised eigenvectors	A1
	$\pm\frac{1}{\sqrt{5}}\begin{pmatrix}0\\-1\\2\end{pmatrix},\ \pm\frac{1}{\sqrt{54}}\begin{pmatrix}1\\-2\\7\end{pmatrix}$ oe	M1: A correct method to normalise at least one of their eigenvectors A1: Both correct. Allow any exact equivalents. Isw	M1 A1
	All marks available regardless of how $a=2,\ \lambda_2=4$ & $\lambda_3=5$ have been obtained		**(5)**
			Total 12