5.

$$\mathbf{A} = \begin{pmatrix} a & a & 1 \\ -a & 4 & 0 \\ 4 & a & 5 \end{pmatrix}$$ where a is a positive constant

(a) Determine the exact value of a for which the matrix $\mathbf{A}$ is singular.

(2)

Given that 2 is an eigenvalue of $\mathbf{A}$

(b) determine

(i) the value of a

(ii) the other two eigenvalues of $\mathbf{A}$

(5)

A normalised eigenvector for the eigenvalue 2 is $\begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix}$

(c) Determine a normalised eigenvector for each of the other eigenvalues of $\mathbf{A}$

(5)

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 12 marks)