8.

$$I_n = \int \cos^n x \, dx \qquad n \geqslant 0$$

(a) Prove that for $n \geqslant 2$

$$I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$$

(4)

(b) Show that for positive even integers n

$$\int_0^{\frac{\pi}{2}} \cos^n x \, dx = \frac{(n-1)(n-3)...5 \times 3 \times 1}{n(n-2)(n-4)...6 \times 4 \times 2} \times \frac{\pi}{2}$$

(4)

(c) Hence determine the exact value of

$$\int_0^{\frac{\pi}{2}} \cos^6 x \sin^2 x \, dx$$

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 11 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA