Question Number	Scheme	Notes	Marks
9(a)(i)	$b^2 = a^2\left(1-e^2\right) \Rightarrow 1 = 9\left(1-e^2\right)$ $\Rightarrow e^2 = ...\left(\frac{8}{9}\right),\ e = \frac{2\sqrt{2}}{3}$ or $\frac{\sqrt{8}}{3}$	M1: Uses a correct eccentricity formula with correct values for a and b and obtains a value for e^2 or e A1: Correct value for e (not $\pm$) Could be implied	M1 A1
	Foci are $\left(\pm 2\sqrt{2}, 0\right)$ or $\left(\pm\sqrt{8}, 0\right)$	B1: Both correct foci as coordinates Condone any use of a negative e **Note that this is not an ft mark.**	B1
			(3)
(a)(ii)	$x = \pm\frac{9\sqrt{2}}{4}$ or $\pm\frac{9\sqrt{8}}{8}$ or $\pm\frac{9}{\sqrt{8}}$ oe Both correct **equations**. Requires single fraction. **Allow ft**: $x = \pm\frac{\mathbf{3}}{\text{their } e}$ computed into a single fraction, condoning e < 0 Allow "$x_1 = ...$, $x_2 = ...$" Condone, e.g., $"x = \pm\frac{a}{e} = \frac{9\sqrt{2}}{4}$ or $-\frac{9\sqrt{2}}{4}"$ but just $"\frac{a}{e} = \pm\frac{9\sqrt{2}}{4}"$ is B0		B1ft
			(2)
(b)	$\lvert PF_1\rvert = e\lvert PM_1\rvert$ or $\lvert PF_2\rvert = e\lvert PM_2\rvert$ oe	States this definition of an ellipse.	M1
Way 1 $PF = ePM$	$\lvert PF_1\rvert + \lvert PF_2\rvert = \underline{e\left(\lvert PM_1\rvert + \lvert PM_2\rvert\right)}$ or $\underline{e\left(\lvert M_1M_2\rvert\right)}$ $"\frac{2\sqrt{2}}{3}" \times 2 \times "\frac{9\sqrt{2}}{4}"$ oe or $\lvert PF_1\rvert + \lvert PF_2\rvert =$ $= "\frac{2\sqrt{2}}{3}"\left("\frac{9\sqrt{2}}{4}" - x\right) + "\frac{2\sqrt{2}}{3}"\left("\frac{9\sqrt{2}}{4}" + x\right)$	Correct method for a numerical expression (or with cancelling "x"s) for $\lvert PF_1\rvert + \lvert PF_2\rvert$ with their e and directrix. **One of the underlined expressions must be seen for the first approach.** **Requires previous M mark.**	**d**M1
	$= 6$ *	Fully correct proof. Modulus signs are not required.	A1*
Way 1 **Guidance**	If they work in a and e, $e \times 2 \times \frac{a}{e}$ is only acceptable if $\underline{e\left(\lvert PM_1\rvert + \lvert PM_2\rvert\right)}$ or $\underline{e\left(\lvert M_1M_2\rvert\right)}$ is seen (as with using the values) and $e\left(\frac{a}{e} - x\right) + e\left(\frac{a}{e} + x\right)\ (\Rightarrow 2a)$ is acceptable but note in both these general cases the second M mark becomes available when $a = 3$ is substituted. **<u>The second M is not available for any work which relies on</u>** $\lvert PF_1\rvert = \lvert PF_2\rvert$ **<u>Their proof needs to be shown to be valid for any position of *P*</u>** So $\lvert PF_1\rvert + \lvert PF_2\rvert = \frac{2\sqrt{2}}{3} \times \frac{9\sqrt{2}}{4} + \frac{2\sqrt{2}}{3} \times \frac{9\sqrt{2}}{4}$ or using $e \times \frac{a}{e} + e \times \frac{a}{e}$ cannot score the second M without $\underline{e\left(\lvert PM_1\rvert + \lvert PM_2\rvert\right)}$ or $\underline{e\left(\lvert M_1M_2\rvert\right)}$ being seen. If e appears as a value it must be correct for the final mark. Just $\lvert PF_1\rvert + \lvert PF_2\rvert = 2a = 2 \times 3 = 6$ is 0/3 Having earned the first mark in Way 1, some candidates proceed to work with a specific point on the ellipse as in Way 2. Further credit is only available if they clearly state e.g, "$\lvert PF_1\rvert + \lvert PF_2\rvert$ is constant for any P"		**(3)**

Question Number	Scheme	Notes	Marks
9(b) **Way 2** $PF_1 + PF_2 = k$	$\lvert PF_1\rvert + \lvert PF_2\rvert = \lvert QF_1\rvert + \lvert QF_2\rvert$ where P and Q are any points on the ellipse oe	States this oe definition of an ellipse, justified by explanation. Accept e.g., "$\lvert PF_1\rvert + \lvert PF_2\rvert$ is constant for any P"	M1
	e.g. Q is where E crosses positive x-axis $\Rightarrow \lvert PF_1\rvert + \lvert PF_2\rvert = 3 - "2\sqrt{2}" + 3 + "2\sqrt{2}"$ Q is where E crosses positive y-axis $\Rightarrow \lvert PF_1\rvert + \lvert PF_2\rvert = 2\sqrt{1^2 + "2\sqrt{2}"^2}$ Q is on E directly above F_1 $\Rightarrow \lvert PF_1\rvert + \lvert PF_2\rvert =$ $\sqrt{1 - \frac{("2\sqrt{2}"^2)}{9}} + \sqrt{(2 \times "2\sqrt{2}")^2 + 1 - \frac{("2\sqrt{2}"^2)}{9}}$	Correct method for a numerical value for $\lvert PF_1\rvert + \lvert PF_2\rvert$ using another point on the ellipse and their foci. **Requires previous M mark.**	**d**M1
	$= 6$ *	Fully correct proof. Modulus signs are not required.	A1*
			(3)
Way 3 **Point in terms of θ**	$P(3\cos\theta, \sin\theta)$ $\lvert PF_1\rvert^2 = (3\cos\theta - "2\sqrt{2}")^2 + \sin^2\theta$ or $\lvert PF_2\rvert^2 = (3\cos\theta + "2\sqrt{2}")^2 + \sin^2\theta$	Correct general point in parametric form and applies Pythagoras for the distance (or its square) to either of their foci. Allow in terms of a, b and θ	M1
	$\lvert PF_1\rvert + \lvert PF_2\rvert =$ $\sqrt{8\cos^2\theta - 12\sqrt{2}\cos\theta + 9} + \sqrt{8\cos^2\theta + 12\sqrt{2}\cos\theta + 9}$	Correct method for $\lvert PF_1\rvert + \lvert PF_2\rvert$ with their foci. Two three term quadratic expressions required but allow the second to be implied if its correct square root is seen. Score when a and b are substituted. **Requires previous M mark.**	**d**M1
	$\lvert PF_1\rvert + \lvert PF_2\rvert =$ $3 - 2\sqrt{2}\cos\theta + 3 + 2\sqrt{2}\cos\theta = 6$*	Fully correct proof. Modulus signs are not required. The intermediate step shown oe is required for this Way.	A1*
			(3)
Way 4 **Point in terms of x**	$P\left(x, \sqrt{1 - \frac{x^2}{9}}\right)$ or $P\left(x, \sqrt{\frac{9 - x^2}{9}}\right)$ $\lvert PF_1\rvert^2 = ("2\sqrt{2}" - x)^2 + 1 - \frac{x^2}{9}$ or $\lvert PF_2\rvert^2 = (x + "2\sqrt{2}")^2 + 1 - \frac{x^2}{9}$	Correct general point in terms of x and applies Pythagoras for the distance (or its square) to either of their foci. Allow in terms of a, b and x.	M1
	$\lvert PF_1\rvert + \lvert PF_2\rvert = \sqrt{\frac{8}{9}x^2 - 4\sqrt{2}x + 9} + \sqrt{\frac{8}{9}x^2 + 4\sqrt{2}x + 9}$	Correct method for $\lvert PF_1\rvert + \lvert PF_2\rvert$ with their foci. Two three term quadratic expressions required but allow the second to be implied if its correct square root is seen. Score when a and b are substituted. **Requires previous M mark.**	**d**M1
	$\lvert PF_1\rvert + \lvert PF_2\rvert = 3 - \frac{2\sqrt{2}}{3}x + 3 + \frac{2\sqrt{2}}{3}x = 6$*	Fully correct proof. Modulus signs are not required. The intermediate step shown oe is required for this Way.	A1*
	Creditworthy alternative approaches will be reviewed		**(3)**

Question Number	Scheme	Notes	Marks
9(c)	$x^2+9(2x+c)^2=9$ or $\frac{x^2}{9}+(2x+c)^2=1$	Substitutes line into the ellipse equation. Condone slips provided intention clear.	M1
	$37x^2+36cx+9c^2-9=0$ or e.g., $\frac{37}{9}x^2+4cx+c^2-1=0$	Correct quadratic with x^2 terms collected (could be implied)	A1
	½ (sum of roots) $\Rightarrow (x=)\frac{-18c}{37}$ or $(x=)\frac{1}{2}\left(\frac{-36c+\sqrt{(36c)^2-4(37)(9c^2-9)}}{2(37)}+\frac{-36c-\sqrt{(36c)^2-4(37)(9c^2-9)}}{2(37)}\right)$ M1: Correct attempt at ½ (sum of roots), i.e., $-\frac{b}{2a}$ for their quadratic. Ignore how the expression is labelled. **Requires previous M mark.** A1: Any correct **equation** in x and c Allow this mark if e.g., x is seen as M_x		**d**M1 A1
	$\Rightarrow c="-\frac{37}{18}"x\Rightarrow y=2x+\left("-\frac{37}{18}"\right)x$ or $x="-\frac{18}{37}"c\Rightarrow y=2\times"-\frac{18}{37}"c+c\Rightarrow\ldots\left(y=\frac{c}{37}\Rightarrow\frac{y}{x}=-\frac{1}{18}\right)$	Substitutes their $c=px$ into the line to obtain an equation in ***x* and *y*** only. Allow e.g., x_M and y_M and condone e.g., suffixes of P & Q This may also be achieved by e.g., finding y in terms of c and then eliminating c with their equation in x and c Must not be using "M_x " or "M_y " etc. but imply this mark from a locus equation in x and y or $x_{\ldots}$ and $y_{\ldots}$ with appropriate suffixes **Requires both previous M marks**	**dd**M1
	$\Rightarrow y_{\ldots}=-\frac{1}{18}x_{\ldots}$ oe $\therefore$ l passes through the origin oe *	Obtains correct equation for locus (accept equivalents) **and** makes conclusion e.g., "passes/goes through origin/O/(0,0)" but allow "shown"/"as required"/"QED" etc. **Requires all previous marks.**	A1*
			(6)
			Total 13
			PAPER TOTAL: 75

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