Question Number	Scheme	Notes	Marks
4	Allow any suitable vector notation throughout this question.		
(a)	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}=\begin{pmatrix}2\\4\\-5\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}\Rightarrow ...$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\0\\-5\end{pmatrix}=\begin{pmatrix}3\\6\\-2\end{pmatrix}.\begin{pmatrix}2\\0\\-5\end{pmatrix}\Rightarrow ...$ $-x+3y+3z=-5$ and $2x-5z=16$	M1: Uses **r**.**n** = **a**.**n** at least once to obtain a plane equation A1: Both correct equations. Accept in **r**.**n** = *p* form	M1 A1
	e.g., $x=\dfrac{16+5z}{2}$	Obtains one variable (may be written as parameter for all marks) in terms of one of the other variables	M1
	$z=\dfrac{2x-16}{5}\Rightarrow x=5+3y+3\left(\dfrac{2x-16}{5}\right)$ $\Rightarrow 5x=25+15y+6x-48\Rightarrow x=-15y+23$ $\left\{x=-15y+23=\dfrac{16+5z}{2}\right\}$	M1: Obtains the variable/parameter in terms of the third variable (or the two other variables in terms of the parameter) A1: Both correct equations	M1 A1 **(M1 on epen)**
	Alternatively, $y=\dfrac{-x+23}{15}=\dfrac{6-z}{6}$ or $z=\dfrac{2x-16}{5}=6-6y$		
	$\left\{\dfrac{x-0}{1}=\dfrac{y-\frac{23}{15}}{-\frac{1}{15}}=\dfrac{z+\frac{16}{5}}{\frac{2}{5}}\Rightarrow\right\}\quad \mathbf{r}=\begin{pmatrix}0\\\frac{23}{15}\\-\frac{16}{5}\end{pmatrix}+\lambda\begin{pmatrix}1\\-\frac{1}{15}\\\frac{2}{5}\end{pmatrix}$ M1: Attempts vector equation of line but "**r** =" may be missing. **Requires all previous M marks.** Allow numerical slips but it must be a correct method i.e., an attempt at $\Rightarrow\dfrac{x-x_1}{l}=\dfrac{y-y_1}{m}=\dfrac{z-z_1}{n}\Rightarrow\mathbf{r}=\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}+\lambda\begin{pmatrix}l\\m\\n\end{pmatrix}$ A1: Any correct equation including "**r** ="		**d**M1 A1
	Or $\left\{\dfrac{x-23}{-15}=\dfrac{y-0}{1}=\dfrac{z-6}{-6}\Rightarrow\right\}\quad \mathbf{r}=\begin{pmatrix}23\\0\\6\end{pmatrix}+\lambda\begin{pmatrix}-15\\1\\-6\end{pmatrix}$ or $\left\{\dfrac{x-8}{\frac{5}{2}}=\dfrac{y-1}{-\frac{1}{6}}=\dfrac{z-0}{1}\Rightarrow\right\}\quad \mathbf{r}=\begin{pmatrix}8\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}\frac{5}{2}\\-\frac{1}{6}\\1\end{pmatrix}$		
	Note that the line may be given in $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=0$ or $\mathbf{r}\times\mathbf{b}=\mathbf{a}\times\mathbf{b}$ form		**(7)**

Question Number	Scheme	Notes	Marks
4(a) **Alt** **Finds point and vector product of normals**	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}=\begin{pmatrix}2\\4\\-5\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}\Rightarrow ...$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\0\\-5\end{pmatrix}=\begin{pmatrix}3\\6\\-2\end{pmatrix}.\begin{pmatrix}2\\0\\-5\end{pmatrix}\Rightarrow ...$ $-x+3y+3z=-5$ and $2x-5z=16$	M1: Uses **r**.**n** = **a**.**n** at least once to obtain a plane equation A1: Both correct equations Accept in **r**.**n** = p form	M1 A1
	e.g., $x=0\Rightarrow z=-\dfrac{16}{5}$	Sets one variable equal to a value and finds a value for another variable. Correct for their equations if no working.	M1
	$3y=-5-3\left(-\dfrac{16}{5}\right)\Rightarrow y=\dfrac{23}{15}$ $\left\{\Rightarrow\left(0,\ \frac{23}{15},\ -\frac{16}{5}\right)\right\}$ Or e.g., $(23,\ 0,\ 6)$, $(8,\ 1,\ 0)$ Points will have the form $(23-15\alpha,\ \alpha,\ 6-6\alpha)$	M1: Proceeds to find a value for the remaining variable. Correct for their equations if no working. A1: Correct values	M1 A1 **(M1 on epen)**
	$\begin{pmatrix}-1\\3\\3\end{pmatrix}\times\begin{pmatrix}2\\0\\-5\end{pmatrix}=...\Rightarrow \mathbf{r}=\begin{pmatrix}0\\\frac{23}{15}\\-\frac{16}{5}\end{pmatrix}+\lambda\begin{pmatrix}-15\\1\\-6\end{pmatrix}$ $\left\{\mathbf{r}=\begin{pmatrix}23\\0\\6\end{pmatrix}+\lambda\begin{pmatrix}-15\\1\\-6\end{pmatrix}\quad \mathbf{r}=\begin{pmatrix}8\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\}$	**d**M1: Attempts vector product of normals (two correct components if method unclear) and forms vector equation with **point** and direction in correct places but allow for a copying error or mix up with components. **Note** that they could obtain the direction from 2 points on the line. **Requires all previous M marks.** "**r** =" may be missing. A1: Any correct equation including "**r** ="	**d**M1 A1
			(7)

Question Number	Scheme	Notes	Marks
4(b)	Note: If 0/5 allow SC 00010 for a correct volume formula seen **<u>for tetrahedron *ABCD*</u>** e.g., $\frac{1}{6}\left\lvert\overrightarrow{CD}.\left(\overrightarrow{CA}\times\overrightarrow{CB}\right)\right\rvert$ Allow with missing modulus but not vector arrows unless implied by further work.		
Way 1 STP inc. $\overrightarrow{CD}$	$\left\lvert\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\rvert=\sqrt{262}\Rightarrow\overrightarrow{CD}=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}$	Attempts magnitude (allow numerical slip) of their direction vector and scales correctly to length 5	M1
	Let C be the point $(8, 1, 0)$ $\overrightarrow{CA}=\begin{pmatrix}2\\4\\-5\end{pmatrix}-\begin{pmatrix}8\\1\\0\end{pmatrix}=...\left\{\begin{pmatrix}-6\\3\\-5\end{pmatrix}\right\}$ and $\overrightarrow{CB}=\begin{pmatrix}3\\6\\-2\end{pmatrix}-\begin{pmatrix}8\\1\\0\end{pmatrix}=...\left\{\begin{pmatrix}-5\\5\\-2\end{pmatrix}\right\}$	Finds vectors for **any** two edges other than *CD*. Could be implied by a distance calculation **if *C* and/or *D* defined**. This mark is not scored if either vector is in terms of a parameter unless it is assigned a value (or is eliminated appropriately) later.	M1
	$\overrightarrow{CD}.\left(\overrightarrow{CA}\times\overrightarrow{CB}\right)=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}.\begin{pmatrix}-6\\3\\-5\end{pmatrix}\times\begin{pmatrix}-5\\5\\-2\end{pmatrix}=...\ \left\{=-\frac{910}{\sqrt{262}}\right\}$	Uses an appropriate scalar triple product with their vectors and finds a value. **Must not include position vectors**. Could be inexact. M0 if clear evidence of an inappropriate method	M1
	$V=\frac{1}{6}\left\lvert\overrightarrow{CD}.\left(\overrightarrow{CA}\times\overrightarrow{CB}\right)\right\rvert=...=\frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	**d**M1: Divides their STP result by 6 and obtains a positive value. Could be inexact. Modulus might not be seen. **Requires previous M mark.** A1: A correct **exact** value	**d**M1 A1
			(5)
Way 2 STP not inc. $\overrightarrow{CD}$	$\left\lvert\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\rvert=\sqrt{262}\Rightarrow\overrightarrow{CD}=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}$	Attempts magnitude (allow numerical slip) of their direction vector and scales correctly to length 5	M1
	Let C be the point $(8, 1, 0)$ $\overrightarrow{AC}=\begin{pmatrix}8\\1\\0\end{pmatrix}-\begin{pmatrix}2\\4\\-5\end{pmatrix}=...\left\{\begin{pmatrix}6\\-3\\5\end{pmatrix}\right\}$ and $\overrightarrow{AB}=\begin{pmatrix}3\\6\\-2\end{pmatrix}-\begin{pmatrix}2\\4\\-5\end{pmatrix}=...\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}\right\}$	Finds vectors for **any** two edges other than *CD*. Could be implied by a distance calculation **if *C* and/or *D* defined**. (See also comment for second M1 in Way 1 re use of a parameter)	M1
	$\overrightarrow{OD}=\begin{pmatrix}8\\1\\0\end{pmatrix}+\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}\Rightarrow\overrightarrow{AD}=\begin{pmatrix}\frac{-75}{\sqrt{262}}+8\\\frac{5}{\sqrt{262}}+1\\\frac{-30}{\sqrt{262}}\end{pmatrix}-\begin{pmatrix}2\\4\\-5\end{pmatrix}=\begin{pmatrix}\frac{-75}{\sqrt{262}}+6\\\frac{5}{\sqrt{262}}-3\\\frac{-30}{\sqrt{262}}+5\end{pmatrix}$ $\Rightarrow\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)=\begin{pmatrix}\frac{-75}{\sqrt{262}}+6\\\frac{5}{\sqrt{262}}-3\\\frac{-30}{\sqrt{262}}+5\end{pmatrix}.\begin{pmatrix}1\\2\\3\end{pmatrix}\times\begin{pmatrix}6\\-3\\5\end{pmatrix}=...\ \left\{=-\frac{910}{\sqrt{262}}\right\}$	Uses an appropriate scalar triple product with their vectors and finds a value. **Must not include position vectors**. Could be inexact. M0 if clear evidence of an inappropriate method	M1
	$V=\frac{1}{6}\left\lvert\overrightarrow{AD}.\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right\rvert=...=\frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	**d**M1: Divides their STP result by 6 and obtains a positive value. Could be inexact. Modulus might not be seen. **Requires previous M mark.** A1: A correct **exact** value	**d**M1 A1
			(5)

Question Number	Scheme	Notes	Marks
4(b) Way 3 Triangle area + perp. distance to plane & vol. of pyramid	$\left\lvert\begin{pmatrix}-15\\1\\-6\end{pmatrix}\right\rvert=\sqrt{262}\Rightarrow\overrightarrow{CD}=\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}$	Attempts magnitude of their direction vector and scales to length 5. See note after next M below.	M1
	Let C be the point $(8, 1, 0)$ $Area\,\Delta ACD=\frac{1}{2}\left\lvert\overrightarrow{CD}\times\overrightarrow{CA}\right\rvert=\frac{1}{2}\left\lvert\frac{5}{\sqrt{262}}\begin{pmatrix}-15\\1\\-6\end{pmatrix}\times\begin{pmatrix}-6\\3\\-5\end{pmatrix}\right\rvert=\ldots\quad\left\{=\frac{65\sqrt{19}}{2\sqrt{262}}\right\}$ Uses formula to find a value for the area of one of the faces. Must be a full method (vector product and modulus). Condone missing $\frac{1}{2}$ Any attempts by trig/Pythagoras must be complete and credible		M1
	Note: It is possible to obtain the area of a relevant triangle such as ACD by e.g., finding the length of the perpendicular distance of point A to the line and multiplying this by $\frac{1}{2}$ x 5 – in such cases allow the first M for completing a viable attempt at the height of the triangle and the second for the area (Condone missing $\frac{1}{2}$)		
	ΔACD is in Π_1 so perp. height of tetrahedron is shortest dist. of $B(3, 6, -2)$ to $-x+3y+3z=-5$: $\left\lvert\frac{-1\times3+3\times6+3\times(-2)+5}{\sqrt{(-1)^2+3^2+3^2}}\right\rvert=\ldots\quad\left\{\frac{14}{\sqrt{19}}\right\}$	Obtains a value for the perpendicular height via formula or any credible method (examples below)	M1
	Parallel planes : $\begin{pmatrix}3\\6\\-2\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}=9,\ \begin{pmatrix}2\\4\\-5\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}=-5\Rightarrow\left\lvert\frac{-5-9}{\sqrt{(-1)^2+3^2+3^2}}\right\rvert=\frac{14}{\sqrt{19}}$ Projection/Resolving : $\overrightarrow{BA}=\begin{pmatrix}1\\2\\3\end{pmatrix}\Rightarrow\frac{\begin{pmatrix}1\\2\\3\end{pmatrix}.\begin{pmatrix}-1\\3\\3\end{pmatrix}}{\sqrt{(-1)^2+3^2+3^2}}=\frac{14}{\sqrt{19}}$		
	$V=\frac{1}{3}\times\frac{65\sqrt{19}}{2\sqrt{262}}\times\frac{14}{\sqrt{19}}=\ldots=\frac{455}{3\sqrt{262}}$ or $\frac{455\sqrt{262}}{786}$	M1: Uses $\frac{1}{3}\times$ area $\Delta\times$ perp. height and obtains a positive value. $\frac{1}{2}$ must have been used for triangle area earlier unless they now use $\frac{1}{6}\times\ldots$ **Requires previous M mark**. A1: Either correct **exact** value	**d**M1 A1
			(5)
			Total 12