Question Number	Scheme	Notes	Marks
5	$\mathbf{M}=\begin{pmatrix}1 & 2 & k\\ -1 & -3 & 4\\ 2 & 6 & -8\end{pmatrix}$		
(i) & (ii) Mark the parts together	$\det\begin{pmatrix}1-\lambda & 2 & k\\ -1 & -3-\lambda & 4\\ 2 & 6 & -8-\lambda\end{pmatrix}$ $=\pm\left[(1-\lambda)\left((-3-\lambda)(-8-\lambda)-24\right)-2\left((-1)(-8-\lambda)-8\right)+k\left((-1)(6)-2(-3-\lambda)\right)\right]$	Recognisable complete attempt at $\det\left(\mathbf{M}-\lambda\mathbf{I}\right)$. May use other rows/columns. Allow $\pm$ and slips including +2 for first –2	M1
	Sarrus $\Rightarrow\pm\left[(1-\lambda)(-3-\lambda)(-8-\lambda)+(2)(4)(2)+(k)(-1)(6)-(k)(-3-\lambda)(2)-(1-\lambda)(4)(6)-(2)(-1)(-8-\lambda)\right]$		
	$=(1-\lambda)\left(\lambda^2+11\lambda\right)-2\lambda+2k\lambda$ $=-\lambda^3-10\lambda^2+9\lambda+2k\lambda$ $=\lambda\left(-\lambda^2-10\lambda+9+2k\right)$	M1: Obtains $\{\lambda\}\left(a\lambda^2+b\lambda+c+dk\text{ oe}\right)$ $a, b, c, d\neq 0$ A1: Correct expression – allow: $\pm\{\lambda\}\left(-\lambda^2-10\lambda+9+2k\text{ oe}\right)$ or $\pm\{\lambda\}\left(\lambda^2+10\lambda-9-2k\text{ oe}\right)$ Allow quadratic to be unsimplified and the marks can be implied if the initial λ has been removed	M1 A1
	{One eigenvalue is zero, if repeated then} $9+2k=0\Rightarrow k=...$ **or** $\left\{\pm\left(-\lambda^2-10\lambda+9+2k\right)\text{has repeated roots so}\right\}$ $b^2-4ac=0\Rightarrow\begin{cases}100-4(-1)(9+2k)=0\\ 100-4(1)(-9-2k)=0\end{cases}\Rightarrow k=...$	Attempts to set their $c+dk=0$ and solves for k **or** Considers the case of their quadratic $a\lambda^2+b\lambda+c+dk=0$ having a repeated root and uses a valid strategy to find k	M1
	Alternative approaches with $\lambda^2+10\lambda-9-2k=0$: $\left(\lambda+a\right)^2=\lambda^2+2a\lambda+a^2\Rightarrow 2a=10\Rightarrow -9-2k=5^2\Rightarrow k=...$ sum of roots $=-10\Rightarrow$ root $=-5\Rightarrow$ product of roots $=(-5)^2=-9-2k\Rightarrow k=...$		
	$k=-\frac{9}{2}$ or $k=-17$	One correct value for k	A1
	{One eigenvalue is zero, if repeated then} $9+2k=0\Rightarrow k=...$ **and** $\left\{\pm\left(-\lambda^2-10\lambda+9+2k\right)\text{has repeated roots so}\right\}$ $b^2-4ac=0\Rightarrow\begin{cases}100-4(-1)(9+2k)=0\\ 100-4(1)(-9-2k)=0\end{cases}\Rightarrow k=...$	Attempts to set their $c+dk=0$ and solves for k **and** Considers the case of their quadratic $a\lambda^2+b\lambda+c+dk=0$ having a repeated root and uses a valid strategy to find k	M1
	$k=-\frac{9}{2}$ with eigenvalue –10 {and 0 repeated} $k=-17$ with eigenvalue –5 {repeated and 0}	Both correct values of k and the associated non-zero eigenvalues clearly assigned. No additional eigenvalues or values for k	A1
			Total 7