6. The ellipse E has equation $\frac{x^2}{16} + \frac{y^2}{9} = 1$

The point $P(4\cos\theta, 3\sin\theta)$ lies on E.

(a) Use calculus to show that an equation of the tangent to E at P is

$$3x\cos\theta + 4y\sin\theta = 12$$

(5)

(b) Determine an equation for the normal to E at P.

(2)

The tangent to E at P meets the x-axis at the point A.
The normal to E at P meets the y-axis at the point B.

(c) Show that the locus of the midpoint of A and B as θ varies has equation

$$x^2(p - qy^2) = r$$

where p, q and r are integers to be determined.

(6)

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Question 6 continued

(Total for Question 6 is 13 marks)