Question Number	Scheme	Notes	Marks
7(a) Way 1	$I_n = \int \cosh^n 2x \ \mathrm{d}x = \int \cosh 2x \cosh^{n-1} 2x \ \mathrm{d}x$ $= \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - \int \frac{1}{2}\sinh 2x \times (n-1)\cosh^{n-2} 2x \times 2\sinh 2x \, \mathrm{d}x$	M1: Correct split and attempts to apply parts to obtain an expression of the correct form (sign and coefficient errors only). A1: Any correct expression	M1 A1
	$\left\{ = \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - (n-1)\int \sinh^2 2x \cosh^{n-2} 2x \, \mathrm{d}x \right\}$ $= \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - (n-1)\int \left(\cosh^2 2x - 1\right)\cosh^{n-2} 2x \, \mathrm{d}x$	Applies $\sinh^2 2x = \pm\cosh^2 2x \pm 1$ **Requires previous M mark.**	**d**M1
	$\Rightarrow I_n = \frac{1}{2}\sinh 2x \cosh^{n-1} 2x - (n-1)\left(I_n - I_{n-2}\right)$	Introduces I_n and I_{n-2} – not implied by given answer. **Requires previous M mark.**	**dd**M1
	$\left\{ \Rightarrow nI_n = \frac{1}{2}\sinh 2x \cosh^{n-1} 2x + (n-1)I_{n-2} \right\}$ $I_n = \frac{\sinh 2x \cosh^{n-1} 2x}{2n} + \frac{n-1}{n} I_{n-2}$ *	Fully correct proof. Condone missing 'dx's. Poor bracketing must be recovered before given answer but no other errors e.g., sin for sinh, or wrong or missing arguments	A1*
	Accept e.g., $I_n = \frac{(n-1)I_{n-2}}{n} + \frac{1}{2n}\sinh 2x \cosh^{n-1} 2x$		**(5)**
Way 2	$I_n = \int \cosh^n 2x \ \mathrm{d}x = \int \cosh^2 2x \cosh^{n-2} 2x \ \mathrm{d}x$ $= \int \left(\sinh^2 2x + 1\right)\cosh^{n-2} 2x \ \mathrm{d}x$	M1: Correct split and applies $\sinh^2 2x = \pm\cosh^2 2x \pm 1$ to obtain an expression of the correct form (sign and coefficient errors only). A1: Correct expression	M1 A1
	$\left\{ = \int \cosh^{n-2} 2x \ \mathrm{d}x + \int \sinh^2 2x \cosh^{n-2} 2x \ \mathrm{d}x \right\}$ $\int \sinh^2 2x \cosh^{n-2} 2x \ \mathrm{d}x \left\{ = \int \sinh 2x \cosh^{n-2} 2x \sinh 2x \ \mathrm{d}x \right\}$ $= \frac{1}{2(n-1)}\sinh 2x \cosh^{n-1} 2x - \frac{1}{n-1}\int \cosh^n 2x \ \mathrm{d}x$	Attempts to apply parts to obtain an expression of the correct form for $\int \sinh^2 2x \cosh^{n-2} 2x \ \mathrm{d}x$ **Requires previous M mark.**	**d**M1
	$\Rightarrow I_n = I_{n-2} + \frac{1}{2(n-1)}\sinh 2x \cosh^{n-1} 2x - \frac{1}{n-1} I_n$	Introduces I_n and I_{n-2} – not implied by given answer. **Requires previous M mark.**	**dd**M1
	$\left\{ \Rightarrow (n-1) I_n = \frac{1}{2}\sinh 2x \cosh^{n-1} 2x + (n-1)I_{n-2} - I_n \right\}$ $I_n = \frac{\sinh 2x \cosh^{n-1} 2x}{2n} + \frac{n-1}{n} I_{n-2}$ *	Fully correct proof. Condone missing 'dx's. Poor bracketing must be recovered before given answer but no other errors e.g., sin for sinh, or wrong or missing arguments	A1*
	Accept e.g., $I_n = \frac{(n-1)I_{n-2}}{n} + \frac{1}{2n}\sinh 2x \cosh^{n-1} 2x$		**(5)**

Question Number	Scheme	Notes	Marks
7(b)	$\left(1+\cosh 2x\right)^3 = 1+3\cosh 2x+3\cosh^2 2x+\cosh^3 2x$ Correct expansion. Could be implied e.g. by $x+3I_1+3I_2+I_3$ and allow if correct but terms are not collected. Condone if partially or completely in "x" provided terms are collected		B1
	$\int \cosh^2 2x\ \mathrm{d}x$ or $I_2=\frac{1}{4}\sinh 2x\cosh 2x+\frac{1}{2}I_0$ or $\int \cosh^3 2x\ \mathrm{d}x$ or $I_3=\frac{1}{6}\sinh 2x\cosh^2 2x+\frac{2}{3}I_1$	Completes an attempt to apply the reduction formula for I_2 **or** I_3. May be slips but must get two terms. May be seen with I_0 / I_1 attempted and/or embedded in expression for $\int\left(1+\cosh 2x\right)^3\mathrm{d}x$	M1
	$I_0=x \qquad I_1=\frac{1}{2}\sinh 2x$ $\int\left(1+\cosh 2x\right)^3\mathrm{d}x=\int\left(1+3\cosh 2x\right)\mathrm{d}x+3I_2+I_3=$ $x+\frac{3}{2}\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{3}{2}x+\frac{1}{6}\sinh 2x\cosh^2 2x+\frac{1}{3}\sinh 2x\left(+c\right)$	$I_0=x$ **and** $I_1=\pm k\sinh 2x$ (condone I_1 from formula) **and** $\int\left(1+3\cosh 2x\right)\mathrm{d}x\to x\pm q\sinh 2x$ **and** uses the above to obtain an expression for $\int\left(1+\cosh 2x\right)^3\mathrm{d}x$ **Requires previous M mark.**	**d**M1
	Note: **One** of I_2 and I_3 may be attempted directly – if so, correct identities must be used and an expression of a correct form obtained. Examples: $I_2=\int\cosh^2 2x\ \mathrm{d}x=\int\left(\frac{1}{2}\cosh 4x+\frac{1}{2}\right)\mathrm{d}x=\frac{1}{8}\sinh 4x+\frac{x}{2}$ $\Rightarrow x+\frac{3}{2}\sinh 2x+\frac{3}{8}\sinh 4x+\frac{3}{2}x+\frac{1}{6}\sinh 2x\cosh^2 2x+\frac{1}{3}\sinh 2x\left(+c\right)$ $I_3=\int\cosh^3 2x\ \mathrm{d}x=\int\cosh 2x\left(\sinh^2 2x+1\right)\mathrm{d}x=\frac{1}{6}\sinh^3 2x+\frac{1}{2}\sinh 2x$ $\Rightarrow x+\frac{3}{2}\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{3}{2}x+\frac{1}{6}\sinh^3 2x+\frac{1}{2}\sinh 2x\left(+c\right)$ If exponential definitions are used they must be correct.		
	$=\frac{5}{2}x+\frac{11}{6}\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{1}{6}\sinh 2x\cosh^2 2x\left(+c\right)$	Correct answer. Award when a correct expression with collected like terms is seen.	A1
	I_2 attempted directly $\Rightarrow\frac{5}{2}x+\frac{11}{6}\sinh 2x+\frac{3}{8}\sinh 4x+\frac{1}{6}\sinh 2x\cosh^2 2x\left(+c\right)$ I_3 attempted directly $\Rightarrow\frac{5}{2}x+2\sinh 2x+\frac{3}{4}\sinh 2x\cosh 2x+\frac{1}{6}\sinh^3 2x\left(+c\right)$		**(4)**
	If identities are used before a correct answer is seen with like terms collected then the work must be correct		**Total 9**