Question Number	Scheme	Notes	Marks
2(a) **Way 1** **TU = I**	$\mathbf{TU = I} \Rightarrow \begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix}\begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\Rightarrow$ e.g., $\begin{matrix} 6a+60-8b=0 \\ -4a-36+5b=1 \end{matrix}$ or $\begin{matrix} -2+3c+7a=0 \\ -3+2c+6a=1 \end{matrix}$ Obtains at least 2 equations with at least one correct. (condone column × row multiplication leading to the way 2 equations – see below). Ignore errors in unused elements or equations.		M1
	e.g., $\begin{matrix} 6a-8b=-60 \\ -4a+5b=37 \end{matrix} \Rightarrow a=\ldots,\ b=\ldots$ **or** $\begin{matrix} 7a+3c=2 \\ 6a+2c=4 \end{matrix} \Rightarrow a=\ldots,\ c=\ldots$ Obtains values for two of a, b and c. You do **not** need to check their values. As long as the previous M mark was scored, it is sufficient to just write down values.		**d**M1
	$a = 2,\ b = 9,\ c = -4$	A1: Two correct values A1: All three correct values and no extra values unless they are rejected.	A1 A1
			(4)
Way 2 **UT = I** **For first 2 marks**	$\mathbf{UT = I} \Rightarrow \begin{pmatrix} 6 & -1 & -4 \\ 15 & c & -9 \\ -8 & a & 5 \end{pmatrix}\begin{pmatrix} 2 & 3 & 7 \\ 3 & 2 & 6 \\ a & 4 & b \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $12-3-4a=1$ $\Rightarrow$ e.g., $42-6-4b=0$ $[45+2c-36=1]$ Obtains at least 2 equations with at least one correct. (condone column × row multiplication leading to the way 1 equations – see above). Ignore errors in unused elements or equations.		M1
	e.g., $-4a = -8,\ -4b = -36\ [2c = -8]$ $\Rightarrow a=\ldots,\ b=\ldots$ Obtains values for two of a, b and c. You do **not** need to check their values. As long as the previous M mark was scored, it is sufficient to just write down values.		**d**M1

Way 3 **Inverses** **For first mark**	$\mathbf{T}^{-1}=\mathbf{U}\Rightarrow\dfrac{1}{4a-5b+36}\begin{pmatrix}2b-24 & -3b+28 & 4\\ 6a-3b & -7a+2b & 9\\ -2a+12 & 3a-8 & -5\end{pmatrix}=\begin{pmatrix}6 & -1 & -4\\ 15 & c & -9\\ -8 & a & 5\end{pmatrix}$ $\Rightarrow$ e.g., $\dfrac{4}{4a-5b+36}=-4,\ \dfrac{2b-24}{4a-5b+36}=6\ \left[\dfrac{-7a+2b}{4a-5b+36}=c\right]$ For $\mathbf{T}^{-1}=\dfrac{1}{\mathrm{f}(a,b)}\mathbf{M}$ where **M** has at least 1 correct element **and** obtains 2 equations. Note that there is no requirement to find all the elements of **M**. **OR** $\mathbf{U}^{-1}=\mathbf{T}\Rightarrow\dfrac{1}{-6a-2c+3}\begin{pmatrix}9a+5c & -4a+5 & 4c+9\\ -3 & -2 & -6\\ 15a+8c & -6a+8 & 6c+15\end{pmatrix}=\begin{pmatrix}2 & 3 & 7\\ 3 & 2 & 6\\ a & 4 & b\end{pmatrix}$ $\Rightarrow$ e.g., $\dfrac{-3}{-6a-2c+3}=3,\ \dfrac{4c+9}{-6a-2c+3}=7\ \left[\dfrac{6c+15}{-6a-2c+3}=b\right]$ For $\mathbf{U}^{-1}=\dfrac{1}{\mathrm{f}(a,c)}\mathbf{M}$ where **M** has at least 1 correct element **and** obtains 2 equations Note that there is no requirement to find all the elements of **M**.	M1

2(b)	$\frac{x-1}{3}=\frac{y}{-4}=z+2\Rightarrow\left[l_2:\mathbf{r}=\right]\begin{pmatrix}1\\0\\-2\end{pmatrix}\pm\lambda\begin{pmatrix}3\\-4\\1\end{pmatrix}$ or $\left(\mathbf{r}-\begin{pmatrix}1\\0\\-2\end{pmatrix}\right)\times\begin{pmatrix}3\\-4\\1\end{pmatrix}=\mathbf{0}$ Obtains parametric/vector form (allow one slip only) or clearly identifies position and direction vectors. May be implied by an attempt to transform both.	M1
	$\begin{pmatrix}6&-1&-4\\15&\text{'}-4\text{'}&-9\\-8&\text{'}2\text{'}&5\end{pmatrix}\begin{pmatrix}1+3\lambda\\-4\lambda\\-2+\lambda\end{pmatrix}=\begin{pmatrix}6+18\lambda+4\lambda+8-4\lambda\\15+45\lambda+16\lambda+18-9\lambda\\-8-24\lambda-8\lambda-10+5\lambda\end{pmatrix}$ **or** their **U** × their $\begin{pmatrix}1&3\\0&-4\\-2&1\end{pmatrix}$ **or** ×their $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ **and** ×their $\begin{pmatrix}3\\-4\\1\end{pmatrix}$ **or** their **U** ×their $\begin{pmatrix}1\\0\\-2\end{pmatrix}$ **and U** ×e.g. $\begin{pmatrix}4\\-4\\-1\end{pmatrix}$ then $dir=\begin{pmatrix}32\\85\\-45\end{pmatrix}-\begin{pmatrix}14\\33\\-18\end{pmatrix}$ Complete and correct method with their *b* and *c* for their **U** × their parametric form or **U** × both vectors or **U** × 2 points on the line and attempts direction. Must be an attempt to mutliply correctly i.e. clearly not row×row but allow attempts that use $\mathbf{T}^{-1}$ for **U** using their *a* and *b* provided all elements are constants and it is a "changed" **T** **OR** $\begin{pmatrix}2&3&7\\3&2&6\\\text{"2"}&4&\text{"9"}\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1+3\lambda\\-4\lambda\\-2+\lambda\end{pmatrix}\Rightarrow\begin{matrix}2x+3y+7z=1+3\lambda\\3x+2y+6z=-4\lambda\\2x+4y+9z=-2+\lambda\end{matrix}$ $x=18\lambda+14$ $\Rightarrow\ y=52\lambda+33$ $z=-18-27\lambda$ A complete method using their parametric form and their **T** to produce and solve 3 simultaneous equations to find *x*, *y* and *z* in terms of λ Alternatively solves $\mathbf{T}\boldsymbol{x}=\left(\text{"}\mathbf{i}-2\mathbf{k}\text{"}\right)$ and $\mathbf{T}\boldsymbol{x}=\left(\text{"}3\mathbf{i}-4\mathbf{k}+\mathbf{k}\text{"}\right)$ to find position and direction	M1
	$\left[l_1:\mathbf{r}=\right]\begin{pmatrix}14+18\lambda\\33+52\lambda\\-18-27\lambda\end{pmatrix}$ $\Rightarrow\frac{x-14}{18}=\frac{y-33}{52}=\frac{z+18}{-27}$	**d**M1: Correctly converts their result into Cartesian equation. **Requires previous method mark** A1: Correct Cartesian equation - allow equivalents e.g., $\ldots=\frac{z-(-18)}{-27}$, $\ldots=\frac{-z-18}{27}$	**d**M1 A1
			(4)
			Total 8

2(b) Alternative

$$x = t \Rightarrow y = \frac{4}{3} - \frac{4}{3}t,\ \ z = \frac{1}{3}t - \frac{7}{3}$$

M1: Obtains parametric form (allow one slip only)

$$\begin{pmatrix} 6 & -1 & -4 \\ 15 & '-4' & -9 \\ -8 & '2' & 5 \end{pmatrix}\begin{pmatrix} t \\ \frac{4}{3} - \frac{4}{3}t \\ \frac{1}{3}t - \frac{7}{3} \end{pmatrix} = \begin{pmatrix} 6t - \frac{4}{3} + \frac{4}{3}t - \frac{4}{3}t + \frac{28}{3} \\ 15t - \frac{16}{3} + \frac{16}{3}t - 3t + 21 \\ -8t + \frac{8}{3} - \frac{8}{3}t + \frac{5}{3}t - \frac{35}{5} \end{pmatrix}$$

M1: As above

$$\left[l_1 : \mathbf{r} = \right]\begin{pmatrix} 8 + 6t \\ \frac{47}{3} + \frac{52}{3}t \\ -9 - 9t \end{pmatrix}$$

$$\Rightarrow \frac{x-8}{6} = \frac{y - \frac{47}{3}}{\frac{52}{3}} = \frac{z+9}{-9}$$

dM1A1: As above