3. The ellipse E has equation

$$\frac{x^2}{49} + \frac{y^2}{b^2} = 1$$

where b is a constant and $0 < b < 7$

The eccentricity of the ellipse is e

(a) Write down, in terms of e only,

(i) the coordinates of the foci of E

(ii) the equations of the directrices of E

(2)

Given that

- the point $P(x, y)$ lies on E where $x > 0$
- the point S is the focus of E on the positive x-axis
- the line l is the directrix of E which crosses the positive x-axis
- the point M lies on l such that the line through P and M is parallel to the x-axis

(b) determine an expression for

(i) PS^2 in terms of e, x and y

(ii) PM^2 in terms of e and x

(2)

(c) Hence show that

$$b^2 = 49\left(1 - e^2\right)$$

(2)

Given that E crosses the y-axis at the points with coordinates $\left(0, \pm 4\sqrt{3}\right)$

(d) determine the value of e

(2)

Given that the x coordinate of P is $\frac{7}{2}$

(e) determine the area of triangle OPM, where O is the origin.

(3)

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)