Question Number	Scheme	Notes	Marks
4(a)	$\mathbf{M}\boldsymbol{x}=\lambda\boldsymbol{x}\Rightarrow\begin{pmatrix}0&-1&3\\-1&4&-1\\3&-1&0\end{pmatrix}\begin{pmatrix}1\\-2\\1\end{pmatrix}=\begin{pmatrix}\lambda\\-2\lambda\\\lambda\end{pmatrix}\Rightarrow$ e.g., $2+3=\lambda\Rightarrow\lambda=5$ **or** $(\mathbf{M}-\lambda\mathbf{I})\boldsymbol{x}=0\Rightarrow\begin{pmatrix}-\lambda&-1&3\\-1&4-\lambda&-1\\3&-1&-\lambda\end{pmatrix}\begin{pmatrix}1\\-2\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\Rightarrow$ e.g., $-\lambda+2+3=0\Rightarrow\lambda=5$ M1: Correct method leading to a value for λ A1: Correct value Note that the working may be minimal so e.g. $2+3=\lambda\Rightarrow\lambda=5$ is sufficient. **Correct answer only scores both marks.**		M1 A1
			(2)
(b)	$\begin{pmatrix}0&-1&3\\-1&4&-1\\3&-1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=-3\begin{pmatrix}x\\y\\z\end{pmatrix}$ **or** $\begin{pmatrix}3&-1&3\\-1&7&-1\\3&-1&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ **or** e.g., $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\-1\\3\end{pmatrix}\times\begin{pmatrix}-1\\7\\-1\end{pmatrix}$ $\Rightarrow x=\ldots, y=\ldots, z=\ldots$ Uses $\mathbf{M}\boldsymbol{x}=-3\boldsymbol{x}$ or $(\mathbf{M}-(-3)\mathbf{I})\boldsymbol{x}=\mathbf{0}$ to produce simultaneous equations **and** obtains values for x, y and z (not all 0) **or** uses a suitable vector product (with two correct components if method unclear)		M1
	$k\begin{pmatrix}1\\0\\-1\end{pmatrix}$	Any correct eigenvector (allow $x=\ldots$, $y=\ldots$, $z=\ldots$ and apply isw if a vector is subsequently formed incorrectly)	A1
			(2)
(c)	$\mathbf{M}\boldsymbol{x}=\lambda\boldsymbol{x}\Rightarrow$ e.g., $-1(1)+3(1)=\lambda$ $(\mathbf{M}-\lambda\mathbf{I})x=0\Rightarrow$ e.g., $-\lambda-1+3=0$ $\lambda=2$ **or** $\lambda^3-4\lambda^2-11\lambda+30=0$ $\det\mathbf{M}=-30=\lambda_1\lambda_2\lambda_3=-15\lambda$ Correct value. May be seen in their **D** which may come from an attempt at $\mathbf{P}^{\mathrm{T}}\mathbf{MP}$.		B1
	$(\mathbf{D}=)\begin{pmatrix}-3&0&0\\0&'2'&0\\0&0&'5'\end{pmatrix}$	Diagonal matrix with –3 and their eigenvalues anywhere on the leading diagonal and 0's elsewhere. Ignore labelling.	B1ft
	$\begin{pmatrix}1\\-2\\1\end{pmatrix}\rightarrow\begin{pmatrix}\frac{\sqrt{6}}{6}\\-\frac{\sqrt{6}}{3}\\\frac{\sqrt{6}}{6}\end{pmatrix}$ or $'\begin{pmatrix}1\\0\\-1\end{pmatrix}'\rightarrow\begin{pmatrix}\frac{\sqrt{2}}{2}\\0\\-\frac{\sqrt{2}}{2}\end{pmatrix}$ or $\begin{pmatrix}1\\1\\1\end{pmatrix}\rightarrow\begin{pmatrix}\frac{\sqrt{3}}{3}\\\frac{\sqrt{3}}{3}\\\frac{\sqrt{3}}{3}\end{pmatrix}$ Correct method seen to normalise **at least one** eigenvector of the two given eigenvectors or their eigenvector from part (b). May be seen in their **P**.		M1
	$\mathbf{D}=\begin{pmatrix}-3&0&0\\0&2&0\\0&0&5\end{pmatrix}$ **and** $\mathbf{P}=\begin{pmatrix}\frac{\sqrt{2}}{2}&\frac{\sqrt{3}}{3}&\frac{\sqrt{6}}{6}\\0&\frac{\sqrt{3}}{3}&-\frac{\sqrt{6}}{3}\\-\frac{\sqrt{2}}{2}&\frac{\sqrt{3}}{3}&\frac{\sqrt{6}}{6}\end{pmatrix}$ Both fully correct, consistent and labelled matrices. Elements may not have had denominators rationalised. (Any columns of **P** could be in opposite direction)		A1
			(4)
			Total 8

Note that some candidates go straight into solving $|\mathbf{M}-\lambda\mathbf{I}|=0$ e.g.

$$\begin{vmatrix} -\lambda & -1 & 3 \\ -1 & 4-\lambda & -1 \\ 3 & -1 & -\lambda \end{vmatrix} = 0 \Rightarrow -\lambda\left(\lambda\left(\lambda-4\right)-1\right)+3+\lambda+3\left(1-3\left(4-\lambda\right)\right)=0$$

$$\Rightarrow \lambda^3-4\lambda^2-11\lambda+30=0 \Rightarrow \lambda=-3,\ 5,\ 2$$

If this is all they do then the B mark in (c) can be awarded for $\lambda = 2$

The other marks in the question are available for the appropriate work.