4.

$$\mathbf{M} = \begin{pmatrix} 0 & -1 & 3 \\ -1 & 4 & -1 \\ 3 & -1 & 0 \end{pmatrix}$$

Given that $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$

(a) determine its corresponding eigenvalue.

(2)

Given that –3 is an eigenvalue of $\mathbf{M}$

(b) determine a corresponding eigenvector.

(2)

Hence, given that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is also an eigenvector of $\mathbf{M}$

(c) determine a diagonal matrix $\mathbf{D}$ and an orthogonal matrix $\mathbf{P}$ such that $\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{MP}$

(4)

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 8 marks)