5. (a) Use the definitions of hyperbolic functions in terms of exponentials to prove that

$$1 - \operatorname{sech}^2 x \equiv \tanh^2 x$$

(3)

$$I_n = \int_0^{\frac{1}{3}\ln 2} \tanh^n 3x \, \mathrm{d}x \qquad n \in \mathbb{Z} \qquad n \geqslant 0$$

(b) Show that

$$I_n = I_{n-2} - \frac{p^{n-1}}{3(n-1)} \qquad n \geqslant 2$$

where p is a rational number to be determined.

(4)

(c) Hence determine the exact value of

$$\int_0^{\frac{1}{3}\ln 2} \tanh^5 3x \, \mathrm{d}x$$

giving your answer in the form $a \ln b + c$ where a, b and c are rational numbers to be found.

(4)

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Question 5 continued

(Total for Question 5 is 11 marks)