Question Number	Scheme/Notes			Marks
7(a)	$y = \arccos(\operatorname{sech} x)$			
	e.g.: $\dfrac{dy}{dx} = -\dfrac{(-\operatorname{sech} x \tanh x)}{\sqrt{1-\operatorname{sech}^2 x}}$	$\cos y = \operatorname{sech} x \Rightarrow$ $-\sin y \dfrac{dy}{dx} = -\operatorname{sech} x \tanh x$ or, e.g., $-\sin y = -\operatorname{sech} x \tanh x \dfrac{dx}{dy}$	$\cos y = (\cosh x)^{-1} \Rightarrow$ $-\sin y \dfrac{dy}{dx} = -(\cosh x)^{-2} \sinh x$	M1
	Differentiates to obtain an equation in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ of the correct form e.g. condone coefficient sign errors only.			
	$\dfrac{dy}{dx} = \dfrac{\operatorname{sech} x \tanh x}{\tanh x}$	$\sqrt{1-\operatorname{sech}^2 x}\dfrac{dy}{dx} = \operatorname{sech} x \tanh x$ $\Rightarrow \tanh x \dfrac{dy}{dx} = \operatorname{sech} x \tanh x$	$\sqrt{1-\operatorname{sech}^2 x}\dfrac{dy}{dx} = \dfrac{\sinh x}{\cosh^2 x}$ $\Rightarrow \tanh x \dfrac{dy}{dx} = \dfrac{\sinh x}{\cosh^2 x}$	**d**M1
	Uses correct identities to obtain an equation in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ in terms of x only with no roots but accept $\sqrt{\tanh^2 x}$ as "no roots"			
	$\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	$\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	$\dfrac{dy}{dx} = \dfrac{\cosh x}{\sinh x} \cdot \dfrac{\sinh x}{\cosh^2 x}$ $\Rightarrow \dfrac{dy}{dx} = \operatorname{sech} x$	A1*
	Fully correct proof. An equation in $\dfrac{dy}{dx}$ or $\dfrac{dx}{dy}$ and exactly two different hyperbolic functions with no roots must be seen before the given answer but accept $\sqrt{\tanh^2 x}$ as "no roots" Withhold this mark for any mathematical error e.g., **clear** use of $\dfrac{d}{dx}(\arccos x) = +\dfrac{1}{\sqrt{1-x^2}}$ and $\dfrac{d}{dx}(\operatorname{sech} x) = +\operatorname{sech} x \tanh x$ or e.g. hyperbolic functions written as trig functions or vice versa. Allow slips if they are recovered but clear and consistent errors score A0			
	Note: There may be other methods seen, e.g., using exponentials and "meeting in the middle"			
				(3)

(b)	e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(\coth x) = -\text{cosech}^2 x$ or $\frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$ or $\frac{-\text{sech}^2 x}{\tanh^2 x}$ or $1-\coth^2 x$ etc. or e.g. $\frac{\left(\mathrm{e}^x - \mathrm{e}^{-x}\right)^2 - \left(\mathrm{e}^x + \mathrm{e}^{-x}\right)^2}{\left(\mathrm{e}^x - \mathrm{e}^{-x}\right)^2}$ or $\frac{2\mathrm{e}^{2x}\left(\mathrm{e}^{2x}-1\right) - 2\mathrm{e}^{2x}\left(\mathrm{e}^{2x}+1\right)}{\left(\mathrm{e}^{2x}-1\right)^2}$ or $\frac{-4}{\left(\mathrm{e}^x - \mathrm{e}^{-x}\right)^2}$ etc. Correct derivative of coth x in any form. Allow recovery if they write e.g. $-\text{cosec}^2 x$ when $-\text{cosech}^2 x$ is clearly implied by subsequent work.	B1
	e.g., sech x – cosech^2x = 0 $\Rightarrow$ sech x = cosech^2x $\Rightarrow \frac{1}{\cosh x} = \frac{1}{\sinh^2 x} \Rightarrow$ $a\cosh^2 x + b\cosh x + c = 0$ or $a\,\text{sech}^2 x + b\,\text{sech}\, x + c = 0$ **or** $\text{sech}\, x - \text{cosech}^2 x = 0 \Rightarrow\Rightarrow \frac{2}{\mathrm{e}^x + \mathrm{e}^{-x}} - \left(\frac{2}{\mathrm{e}^x - \mathrm{e}^{-x}}\right)^2 = 0 \Rightarrow$ $\Rightarrow A\mathrm{e}^{4x} + B\mathrm{e}^{3x} + C\mathrm{e}^{2x} + D\mathrm{e}^x + E = 0$ Sets $\mathrm{f}'(x) = 0$ **and** uses correct identities to obtain a 3TQ in cosh x or sech x **or** substitutes the correct exponential forms and obtains a 5 term quartic in e^x	M1
	$\cosh^2 x - \cosh x - 1 = 0$ or $\text{sech}^2 x + \text{sech}\, x - 1 = 0$ oe **or** $\Rightarrow \mathrm{e}^{4x} - 2\mathrm{e}^{3x} - 2\mathrm{e}^{2x} - 2\mathrm{e}^x + 1 = 0$ oe Correct quadratic equation or correct quartic equation.	A1
	$\cosh x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$ $\left(= \frac{1+\sqrt{5}}{2}\right)$ or e.g., $\left(\text{sech}\, x + \frac{1}{2}\right)^2 - \frac{1}{4} - 1 = 0 \Rightarrow \text{sech}\, x = \left(\frac{-1+\sqrt{5}}{2}\right)$ Solves quadratic resulting from sech x + their derivative of coth x = 0 Must obtain a **real and exact** value > 1 (or between 0 and 1 if sech used). Apply usual rules. (No need to reject invalid values) If no solving method seen one solution must be consistent with their equation. For the 5 term quartic in e^x progress is unlikely unless they proceed via e.g. $\left(\mathrm{e}^{2x} - \left(1+\sqrt{5}\right)\mathrm{e}^x + 1\right)^2 = 0$	**d**M1
	$x = \text{arcosh}\left(\frac{1+\sqrt{5}}{2}\right) = \ln\left(\frac{1+\sqrt{5}}{2} + \sqrt{\left(\frac{1+\sqrt{5}}{2}\right)^2 - 1}\right)$ **or** $\frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2} = \frac{1+\sqrt{5}}{2} \Rightarrow \mathrm{e}^{2x} - \left(1+\sqrt{5}\right)\mathrm{e}^x + 1 = 0 \Rightarrow \mathrm{e}^x = \frac{1+\sqrt{5} + \sqrt{\left(1+\sqrt{5}\right)^2 - 4}}{2} \Rightarrow x = \ldots$ Uses correct logarithmic form or exponentials to find x as a ln of an exact value. Exponential definition must be correct and quadratic solving subject to usual rules or consistent with their equation leading to a value of $\mathrm{e}^x > 0$	**dd**M1
	$\Rightarrow x = \ln\left(\frac{1}{2}\left(1+\sqrt{5}\right) + \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\right)$ or accept $x = \ln\left(\frac{1+\sqrt{5}}{2} + \sqrt{\frac{1+\sqrt{5}}{2}}\right)$ Note that $x = \ln\frac{1}{2}\left(1+\sqrt{5}\right) + \sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}$ scores A0	A1
		(6)
		Total 9

Correct work in (b) leading to:

$$\cosh^2 x - \cosh x - 1 = 0 \Rightarrow \cosh x = \frac{1+\sqrt{5}}{2}$$

$$x = \operatorname{arcosh}\left(\frac{1+\sqrt{5}}{2}\right) = \ln\left(\frac{1+\sqrt{5}}{2} + \sqrt{\frac{1+\sqrt{5}}{2}}\right)$$

With no evidence where the $\sqrt{\frac{1+\sqrt{5}}{2}}$ comes from, scores: B1M1A1dM1ddM0A0