Question Number	Scheme	Notes	Marks
2(a)	$\det(\mathbf{M}-\lambda\mathbf{I})=\begin{vmatrix}2-\lambda & 0 & 3\\ 0 & -4-\lambda & -3\\ 0 & -4 & -\lambda\end{vmatrix}$ = e.g., $(2-\lambda)\left[(-4-\lambda)(-\lambda)-(-4)(-3)\right]-0+3(0)$ or $(2-\lambda)\left[(-4-\lambda)(-\lambda)-(-4)(-3)\right]-0+0$ Sarrus $\Rightarrow(2-\lambda)(-4-\lambda)(-\lambda)-(2-\lambda)(-3)(-4)$	Obtains an unsimplified cubic expression for $\det(\mathbf{M}-\lambda\mathbf{I})$ condoning sign/copying slips only. Allow poor bracketing if intention clear.	**M1**
	Note: It is possible to just use $\mathbf{M}\boldsymbol{x}=\lambda\boldsymbol{x}$ e.g., $-4y=\lambda z\Rightarrow y=-\frac{\lambda z}{4}$ and $-4y-3z=\lambda y\Rightarrow\lambda z-3z=-\frac{\lambda^2 z}{4}\Rightarrow\lambda^2+4\lambda-12=0\Rightarrow...$ Score the M1 for achieving a 3TQ in λ from appropriate work condoning copying/sign slips only		
	$(2-\lambda)(\lambda^2+4\lambda-12)=0$ or $\lambda^3+2\lambda^2-20\lambda+24=0$ or $-\lambda^3-2\lambda^2+20\lambda-24=0$ $(2-\lambda)(\lambda-2)(\lambda+6)=0$ or $(\lambda+6)(\lambda-2)(\lambda-2)=0$ $\lambda_1=-6\ \ (\lambda_2=2)$ **d**M1: Solves $\det(\mathbf{M}-\lambda\mathbf{I})=0$ to obtain any value for λ including 2. Not usual rules – award for any value seen that is consistent with their equation. The "=0" can be implied by a solution. Note that they may disregard the $(2-\lambda)$ and solve a quadratic. A1: –6 from a correct equation. Accept both solutions e.g., "– 6, 2" and allow if mislabelled and/or –6 rejected. No incorrect solutions.		**dM1 A1**
	$\mathbf{M}\boldsymbol{x}=-6\boldsymbol{x}\ \Rightarrow\ \begin{matrix}2x+3z=-6x\\ -4y-3z=-6y\\ -4y=-6z\end{matrix}\ \Rightarrow x=...,\ y=...,\ z=...$ $(\mathbf{M}+6\mathbf{I})\boldsymbol{x}=\mathbf{0}\ \Rightarrow\ \begin{matrix}8x+3z=0\\ 2y-3z=0\\ -4y+6z=0\end{matrix}\ \Rightarrow x=...,\ y=...,\ z=...$	Uses $\mathbf{M}\boldsymbol{x}=\lambda\boldsymbol{x}$ or $(\mathbf{M}-\lambda\mathbf{I})\boldsymbol{x}=0$ with any of their non-zero eigenvalues (however obtained) to form simultaneous equations and solves. No requirement for a vector for this mark. There is no need to check their values but award M0 for a zero solution.	**M1**
	Note: Could find vector product of first 2 rows of $\mathbf{M}-\lambda\mathbf{I}$ i.e., $(8\mathbf{i}+3\mathbf{k})\times(2\mathbf{j}-3\mathbf{k})=(-6\mathbf{i}+24\mathbf{j}+16\mathbf{k})$ (two correct components)		
	$\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-3\\12\\8\end{pmatrix}\Rightarrow\frac{1}{\sqrt{3^2+12^2+8^2}}\begin{pmatrix}-3\\12\\8\end{pmatrix}$	Correct method to normalise their eigenvector no matter how this vector is obtained provided it has at least 2 non-zero components. Only allow slips if there is working.	**M1**
	e.g., $\frac{1}{\sqrt{217}}\begin{pmatrix}-3\\12\\8\end{pmatrix}$ or $\begin{pmatrix}-\frac{3\sqrt{217}}{217}\\ \frac{12\sqrt{217}}{217}\\ \frac{8\sqrt{217}}{217}\end{pmatrix}$ or $\begin{pmatrix}-\frac{3}{\sqrt{217}}\\ \frac{12}{\sqrt{217}}\\ \frac{8}{\sqrt{217}}\end{pmatrix}$ or $\frac{1}{2\sqrt{217}}\begin{pmatrix}-6\\24\\16\end{pmatrix}$	A correct normalised eigenvector in any form. Note direction may be reversed. May use **i**, **j**, **k** notation	**A1**
			(6)

Question Number	Scheme	Notes	Marks
2(b)	May use **i**, **j**, **k** notation		
	Multiplies position and direction by **M** (not e.g., $\mathbf{M}-\lambda\mathbf{I}$) In parametric form: $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}4+2\mu\\-1\\-\mu\end{pmatrix}=\ldots \quad \left\{\begin{pmatrix}8+4\mu-3\mu\\4+3\mu\\4\end{pmatrix}=\begin{pmatrix}8\\4\\4\end{pmatrix}+\mu\begin{pmatrix}1\\3\\0\end{pmatrix}\right\}$ There is no requirement to extract the vectors if parametric form is used. Allow this mark if e.g., $8+4\mu-3\mu$ written as $2(4+2\mu)-3\mu$ Allow this work without a parameter i.e., $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}4\\-1\\0\end{pmatrix}=\ldots \quad \left\{\begin{pmatrix}8\\4\\4\end{pmatrix}\right\}$ **and** $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}2\\0\\-1\end{pmatrix}=\ldots \quad \left\{\begin{pmatrix}1\\3\\0\end{pmatrix}\right\}$ **or** $\begin{pmatrix}2&0&3\\0&-4&-3\\0&-4&0\end{pmatrix}\begin{pmatrix}4&2\\-1&0\\0&-1\end{pmatrix}=\ldots \quad \left\{\begin{pmatrix}8&1\\4&3\\4&0\end{pmatrix}\right\}$ Alternatively: Could find 2 points on l_1, transform them both and **subtract** to find direction. Allow slips and condone the matrix product written the wrong way round provided they have attempted to multiply the elements appropriately and they obtain a vector (or 3 x 2 matrix) with the resulting values correctly placed. Condone if they proceed to confuse which is the position and which is the direction.		**M1**
	$\mathbf{r}\times\begin{pmatrix}1\\3\\0\end{pmatrix}=\begin{pmatrix}8\\4\\4\end{pmatrix}\times\begin{pmatrix}1\\3\\0\end{pmatrix}$	Forms: $\mathbf{r}\times$ direction = position $\times$ direction Must not clearly confuse their vectors. Allow if RHS = direction x position. **Requires previous M mark.** No requirement to calculate vector product but the RHS could be implied by 2 correct components (or the negative version if the product is reversed)	**dM1**
	$\mathbf{r}\times\begin{pmatrix}1\\3\\0\end{pmatrix}=\begin{pmatrix}-12\\4\\20\end{pmatrix}$	Any correct equation in the correct form. Not $\mathbf{b}=\ldots$, $\mathbf{c}=\ldots$ unless $\mathbf{r}\times\mathbf{b}=\mathbf{c}$ seen. Isw once a correct answer is seen.	**A1**
			(3)
			Total 9