Question Number	Scheme	Notes	Marks
3(a)	$y = \text{arsinh}\left(\sqrt{x^2-1}\right)$		
	For all Ways allow the final answer to be written as $\dfrac{1}{\left(x^2-1\right)^{\frac{1}{2}}}$ or $\left(x^2-1\right)^{-\frac{1}{2}}$		
Way 1	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{1+\left(\sqrt{x^2-1}\right)^2}} \times \dfrac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}(2x)$ M1: Obtains $\dfrac{1}{\sqrt{1+\left(\sqrt{x^2-1}\right)^2}} \times \mathrm{f}\left(x\right)$ or e.g., $\dfrac{1}{x} \times \mathrm{f}\left(x\right)$ $\quad \mathrm{f}\left(x\right) \neq k$ A1: Fully correct unsimplified expression		**M1** **A1**
	$= \dfrac{1}{\sqrt{1+x^2-1}} \times \dfrac{x}{\sqrt{x^2-1}} = \dfrac{1}{\sqrt{x^2-1}}*$ or e.g., $= \dfrac{1}{x} \times \dfrac{x}{\sqrt{x^2-1}} = \dfrac{1}{\sqrt{x^2-1}}*$	Correct completion with intermediate line of working and no errors	**A1***
			(3)
Way 2 **Takes sinh of both sides**	$y = \text{arsinh}\left(\sqrt{x^2-1}\right) \Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \cosh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}(2x)$ M1: Takes sinh of both sides and differentiates to obtain $\cosh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}\left(x\right)$ $\quad \mathrm{f}(x) \neq k$ A1: Fully correct unsimplified equation		**M1** **A1**
	$\cosh y = \sqrt{1+\sinh^2 y}$ or $\sqrt{1+x^2-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{x^2-1}}*$	Correct completion with clear use of identity (must see more than just $\cosh y = x$) and no errors	**A1***
			(3)
Way 3 **Takes sinh & squares**	$y = \text{arsinh}\left(\sqrt{x^2-1}\right) \Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \sinh^2 y = x^2-1 \Rightarrow 2\sinh y \cosh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x$ M1: Takes sinh of both sides, squares and differentiates to obtain $c\sinh y \cosh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}\left(x\right)$ $\quad \mathrm{f}(x) \neq k$ A1: Fully correct unsimplified expression or equation		**M1** **A1**
	$\cosh y = \sqrt{1+\sinh^2 y}$ or $\sqrt{1+x^2-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{x^2-1}}$	Correct completion with clear use of identity (must see more than just $\cosh y = x$) and no errors	**A1***
			(3)
Way 4 **Takes sinh & squares & uses identity**	$\Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \sinh^2 y = x^2-1 \Rightarrow \cosh^2 y = 1+\left(x^2-1\right) \Rightarrow \cosh^2 y = x^2 \Rightarrow 2\sinh y \cosh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x$ M1: Takes sinh of both sides, squares, uses identity and differentiates to obtain $c\sinh y \cosh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$ $\quad \mathrm{f}\left(x\right) \neq k$ Allow sign errors with identity for the M mark. A1: Fully correct unsimplified expression or equation		**M1** **A1**
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{x^2-1}}$	Correct completion with clear use of identity and no errors	**A1***
			(3)

Question Number	Scheme	Notes	Marks
3(a) Way 5 **Takes sinh & squares & uses identity & roots**	$\Rightarrow \sinh y = \sqrt{x^2-1} \Rightarrow \sinh^2 y = x^2-1 \Rightarrow \cosh^2 y = 1+\left(x^2-1\right) \Rightarrow \cosh y = x \Rightarrow \sinh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ M1: Takes sinh of both sides, squares, uses identity, roots and differentiates to obtain $c\sinh y \dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$ or k Allow sign errors with identity. A1: Fully correct unsimplified expression or equation		**M1** **A1**
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sqrt{x^2-1}}$	Correct completion with clear use of identity and no errors	**A1***
			(3)
Way 6 **Uses log form of arsinh first**	$y = \operatorname{arsinh}\left(\sqrt{x^2-1}\right) \Rightarrow y = \ln\left(\sqrt{x^2-1}+\sqrt{x^2-1+1}\right) = \ln\left(\sqrt{x^2-1}+x\right) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{1}{2}\left(x^2-1\right)^{-\frac{1}{2}}(2x)+1}{\sqrt{x^2-1}+x}$ M1: Use log form of arsinh correctly and differentiates to obtain $\dfrac{\mathrm{f}(x) \neq k}{\sqrt{x^2-1}+x}$ A1: Fully correct unsimplified expression		**M1** **A1**
	$= \dfrac{\frac{x}{\sqrt{x^2-1}}+1}{\sqrt{x^2-1}+x}$ or $\dfrac{x+\sqrt{x^2-1}}{\sqrt{x^2-1}} \times \dfrac{1}{\sqrt{x^2-1}+x} = \dfrac{1}{\sqrt{x^2-1}}$ *	Correct completion with intermediate line of working and no errors	**A1***
			(3)
You may see other variations e.g., using exponential definitions, attempts via dx/dy. The M mark is for differentiating to obtain correct forms and the first A is awarded if it is correct. The final A is for correct completion.			

Question Number	Scheme	Notes	Marks
3(b)	$\mathrm{f}(x)=\frac{1}{3}\operatorname{arsinh}\left(\sqrt{x^2-1}\right)-\arctan x$		
	$\mathrm{f}'(x)=\frac{1}{3\sqrt{x^2-1}}-\frac{1}{1+x^2}$	$\mathrm{f}'(x)=\frac{A}{\sqrt{x^2-1}}\pm\frac{1}{1\pm x^2}$ $A=\frac{1}{3}$, 3 or 1	**M1 (B1 on ePen)**
	$1+x^2=3\sqrt{x^2-1}$ $1+2x^2+x^4=9x^2-9$	Sets $\frac{A}{\sqrt{x^2-1}}\pm\frac{1}{1+x^2}=0$ $A=\frac{1}{3}$, 3 or 1 **Denominator of derivative of arctan *x* must now be 1 + x^2** Cross multiplies and squares to obtain the correct form for both sides so do not condone e.g., $\left(1+x^2\right)^2=1+x^4$ May see the quartic obtained through equivalent work.	**M1**
	$x^4-7x^2+10=0\Rightarrow\left(x^2-2\right)\left(x^2-5\right)=0\Rightarrow x^2=2,\ 5$ Solves a 3TQ in x^2 (usual rules and one correct root if no working). No requirement to see the terms collected. Ignore labelling of solutions so allow e.g., "$\underline{x}=2,\ 5$". One correct value for their equation if no working, which may be for x or x^2, so just look for the values. May change the variable. Allow for a correct solution with no working from solving a three term quartic of the correct form on a calculator. Allow if value for x^2 is negative or if roots are complex. **Requires previous M marks.**		**ddM1**
	$x=\sqrt{2},\ \sqrt{5}$	Both exact and no other solutions e.g., $\pm$ is A0 unless negatives rejected. Must not reject either correct solution.	**A1**
			(4)
			Total 7