Question Number	Scheme/Notes	Marks
4(a)	$\sinh(A+B)=\sinh A\cosh B+\cosh A\sinh B$	
	There is no credit for proofs that do not use exponential definitions	
	$\{\sinh A\cosh B+\cosh A\sinh B=\}$ $\frac{e^A-e^{-A}}{2}\times\frac{e^B+e^{-B}}{2}+\frac{e^A+e^{-A}}{2}\times\frac{e^B-e^{-B}}{2}$ or e.g., $\frac{\left(e^A-e^{-A}\right)\left(e^B+e^{-B}\right)+\left(e^A+e^{-A}\right)\left(e^B-e^{-B}\right)}{4}$ Replaces two of the four hyperbolic functions with correct exponential expressions. Condone poor bracketing. If they immediately start expanding this mark is only implied by completely correct work (i.e., with exponential definitions correct) and not just the fractions shown in the A1* note	**M1**
	$=\frac{e^{A+B}-e^{B-A}+e^{A-B}-e^{-A-B}+e^{A+B}+e^{B-A}-e^{A-B}-e^{-A-B}}{4}$ Expands numerator (or numerators if 2 separate fractions). Allow for sign errors only with coefficients and indices **and must see at least four terms** (but count terms which have been crossed out by cancelling) Allow this mark for: $=\frac{e^Ae^B-e^{-A}e^B+e^Ae^{-B}-e^{-A}e^{-B}+e^Ae^B+e^{-A}e^B-e^Ae^{-B}-e^{-A}e^{-B}}{4}$ Must see at least four terms as before but the last mark will not be available unless the requirements shown below are satisfied.	**M1**
	$=\frac{2e^{A+B}-2e^{-(A+B)}}{4}$ or $\frac{2\left(e^{A+B}-e^{-(A+B)}\right)}{4}$ or $\frac{e^{A+B}-e^{-(A+B)}}{2}$ or $\frac{1}{2}\left(e^{A+B}-e^{-(A+B)}\right)$ or $\frac{e^{A+B}}{2}-\frac{e^{-(A+B)}}{2}$ $=\sinh(A+B)$* Reaches $\sinh(A+B)$ with no errors. Condone if the "$\sinh A\cosh B+\cosh A\sinh B=$" is missing at the start but the "$=\sinh(A+B)$" or "= LHS" must be seen. All bracketing correct where required but condone an unclosed bracket. One of the expressions shown or similar must be seen and allow –*A*–*B* used for $-(A+B)$. Allow a "meet in the middle" proof and condone a "1=1" style approach provided it is complete. In both these cases a minimal conclusion is required e.g., "shown" but allow if both "LHS = ..." and "...=RHS" are seen. Do not condone sinh and/or cosh written as sin/cos for this mark	**A1***
	Attempts that start with the LHS and do not revert to a "meet in the middle" approach: Score the second M provided an **eight** term expanded numerator is achieved. The first M is for two explicitly clear correct replacements of hyperbolic expressions with two of $\sinh A, \cosh B, \cosh A$ and $\sinh B$. Condone if the $\sinh(A+B)=$ is missing at the start in these cases but the RHS or "...=RHS" must be seen.	
		(3)

Question Number	Scheme	Notes	Marks
4(b)	Condone the use of e.g., B for α or k for R for the first three marks but allow the A mark if recovered which may be via a correct expression which might be in (c)		
	$10\sinh x+8\cosh x = R\sinh x\cosh\alpha + R\cosh x\sinh\alpha$ $\Rightarrow R\sinh\alpha = 8,\quad R\cosh\alpha = 10$ Equates coefficients to obtain the two correct equations. This mark could be implied by either **correct** elimination, i.e., $R^2 = 10^2 - 8^2$ or $\tanh\alpha = \frac{8}{10}$ **provided incorrect equations are not seen.**		**B1 (M1 on ePen)**
	A complete attempt at finding a positive value for R: **By elimination**: $R^2\left(\cosh^2\alpha - \sinh^2\alpha\right) = 10^2 - 8^2 \Rightarrow R^2 = 36 \Rightarrow R = 6$ Allow this mark for $R = \sqrt{10^2+8^2} = 2\sqrt{41}$ or $\sqrt{164}$. May just see e.g., $R = 2\sqrt{41}$ **Following a positive value obtained for α where $\alpha = k\ln p,\ k>0,\ p>1$:** $\alpha = \frac{1}{2}\ln 9 = \ln 3 \Rightarrow R\cosh(\ln 3) = 10 \Rightarrow R\left(\frac{e^{\ln 3}+e^{-\ln 3}}{2}\right) = 10 \Rightarrow R = \ldots\ \left\{\frac{5}{3}R = 10 \Rightarrow R = 6\right\}$ or $R\sinh(\ln 3) = 8 \Rightarrow R\left(\frac{e^{\ln 3}-e^{-\ln 3}}{2}\right) = 8 \Rightarrow R = \ldots\ \left\{\frac{4}{3}R = 8 \Rightarrow R = 6\right\}$ Correct exponential definitions must be used but can be implied by correct work. Allow if the 10 and 8 are mixed up and allow slips in solving		**1st M1**
	A complete attempt at finding a positive value for α where $\alpha = k\ln p,\ k>0,\ p>1$: **By elimination:** $\tanh\alpha = \frac{8}{10} \Rightarrow \alpha = \text{artanh}\left(\frac{4}{5}\right) = \frac{1}{2}\ln\left(\frac{1+\frac{4}{5}}{1-\frac{4}{5}}\right) = \ldots\ \left\{= \frac{1}{2}\ln 9 = \ln 3\right\}$ A correct logarithmic form must be used with a valid value for artanh (<1) **Following a positive value obtained for *R*:** $\sinh\alpha = \frac{8}{"6"} \Rightarrow \alpha = \text{arsinh}\left(\frac{8}{"6"}\right) = \ln\left(\frac{8}{"6"} + \sqrt{\left(\frac{8}{"6"}\right)^2+1}\right)\ \{=\ln 3\}$ $\cosh\alpha = \frac{10}{"6"} \Rightarrow \alpha = \text{arcosh}\left(\frac{10}{"6"}\right) = \ln\left(\frac{10}{"6"} + \sqrt{\left(\frac{10}{"6"}\right)^2-1}\right)\ \{=\ln 3\}$ A correct logarithmic form must be used with a valid value if using arcosh (>1) The appropriate logarithmic forms could be implied by correct values. Allow this mark if e.g., $\frac{8}{10}$ is erroneously simplified but the value must be valid for the inverse hyperbolic function. If an exponential form is used to evaluate an inverse hyperbolic the form must be correct and the solving of any resulting 3TQ (most likely in e^{α} or e^{x}) must satisfy usual rules with one root correct if no working. Note that using tanh leads to a 2TQ which they must get one correct root for. They must also proceed to $\alpha = k\ln p,\ k>0,\ p>1$		**2nd M1**
	$6\sinh(x+\ln 3)$ or $R=6$ and $\alpha = \ln 3$ (or $p=3$) Correct expression but allow values for R and α (or p). If all the values are not seen in (b) then allow if they are seen in (c) and they could be seen embedded in a correct expression. A0 for additional solutions e.g., $6\sinh(x\pm\ln 3)$		**A1**

Question Number	Scheme/Notes	Marks
4(c)	There is no credit for attempts that do not use part (b) so e.g., do not award marks for attempts that apply exponential definitions to $10\sinh x+8\cosh x=18\sqrt{7}$ but note that it is acceptable to use exponential definitions with $6\sinh\left(x+\ln 3\right)=18\sqrt{7}$. Allow work with "made up" values for R and p provided $R>0,\ p\in\mathbb{Z},\ p>1$	
	$6\sinh\left(x+\ln 3\right)=18\sqrt{7}$ $\Rightarrow x=\operatorname{arsinh}\left(3\sqrt{7}\right)-\ln 3$ $\Rightarrow x=\ln\left(3\sqrt{7}+\sqrt{\left(3\sqrt{7}\right)^2+1}\right)-\ln 3$ Obtains $x=\operatorname{arsinh}\left(\frac{18\sqrt{7}}{"6"}\right)\pm\ln"3"$ or $x\pm\ln"3"=\operatorname{arsinh}\left(\frac{18\sqrt{7}}{"6"}\right)$ from $"6"\sinh\left(x\pm\ln"3"\right)=18\sqrt{7}$ **and** uses the correct logarithmic form to obtain an expression for, or equation in x in "ln"s only but condone loss of the –ln "3" or +ln"3" after it has been seen. If the –ln "3" or +ln"3" is immediately incorporated to make a single logarithm the subtraction/addition law must be applied correctly. Work must be exact and not in decimals. If e.g., $C=\operatorname{arsinh}\left(3\sqrt{7}\right)$ is found using $\frac{e^C-e^{-C}}{2}=3\sqrt{7}$, the exponential definition must be correct and they must solve a 3TQ in e^C satisfying usual rules (or one root correct if no working) and proceed to a valid $C=...$ (e.g., not ln(negative)). This also applies to attempts via $6\frac{e^{x+\ln 3}-e^{-x-\ln 3}}{2}=18\sqrt{7}\quad\left\{\Rightarrow 3e^x-\frac{1}{3}e^{-x}=6\sqrt{7}\Rightarrow 9e^{2x}-18\sqrt{7}e^x-1=0\Rightarrow x=\ln\left(\frac{8+3\sqrt{7}}{3}\right)\right\}$ Note that $e^{2(x+\ln 3)}-6\sqrt{7}e^{x+\ln 3}-1=0\Rightarrow e^{x+\ln 3}=8+3\sqrt{7}\Rightarrow x=\ln\left(\frac{8+3\sqrt{7}}{3}\right)$ is also possible and in such cases the x + ln "3" must be handled correctly	**M1**
	$\left\{x=\ln\left(\frac{3\sqrt{7}+8}{3}\right)=\right\}\ln\left(\sqrt{7}+\frac{8}{3}\right)$ Correct answer in correct form. Accept e.g., $\ln\left(2\frac{2}{3}+\sqrt{7}\right)$. Must be fully bracketed correctly. Accept $q=\frac{8}{3}$ if $\ln\left(\sqrt{7}+q\right)$ is seen. No additional answers.	**A1**
		(2)
		Total 9